Understanding the contributions of working memory components over the primary school years to enable screening for future attainment in mathematics

ALLEN, KATIE

Understanding the contributions of working memory components over the primary school years to enable screening for future attainment in mathematics

Katie Allen

A thesis presented for the degree of

Doctor of Philosophy

School of Education

The University of Durham

United Kingdom

September 2020

Declaration
Some material in this thesis is based on joint research with Dr David Giofrè, currently at the University of Genoa, Italy. The following lists Dr David Giofrè's contributions to each paper included:

Study 1, papers 1 and 2: Support with learning analysis methods and collaborative writing of the paper.
Study 2: Advice on precise measures, support with analysis, and collaborative writing of the paper.
Study 4: Support with analysis and collaborative writing of the paper.

All other included work is the author's own.

Acknowledgements
I would like to acknowledge the Economic and Social Research Council and the Northern Ireland and North East Doctoral Training Partnership for their support in funding this PhD research project. I would also like to acknowledge the support of my supervisors, Professor Steve Higgins and Dr John Adams, throughout, and Dr David Giofrè for his contributions to the published papers included.

Table of Contents

List of Tables

List of Figures

Abstract

Interest in, and evidence for, the involvement of working memory in mathematics is increasing as the performance of school leavers is under constant scrutiny. Understanding how components of working memory relate to aspects of mathematics is, however, limited. The stability of this relationship when other cognitive predictors are included is not known, nor is whether the relationship is stable over time. This thesis contains a systematic review of the literature and four studies investigating relationships between working memory and mathematics performance.

As a first step, available literature on the relationship between visuospatial working memory and mathematics performance was reviewed in a systematic, thematic analysis of effect sizes. Results showed a significant influence on the effect size of standardised mathematics measures, but not the type of visuospatial working memory or mathematics being assessed. Crucially, the overall effect size was positive, demonstrating a positive association between visuospatial working memory and mathematics performance.

The first study built on these findings to identify the relative contributions of verbal, spatial-simultaneous, and spatial-sequential working memory in written mathematics. Year 3 children (7-8 years of age, n=214) in the UK completed a battery of working memory tasks alongside a standardised mathematics test. Results showed the largest individual contribution was from verbal working memory, followed by spatial-simultaneous factors. This suggests the components of working memory underpinning mathematical performance at this age are verbal-numeric and spatial-simultaneous. The study raised the question of whether this relationship is consistent across the primary school years.

The second study therefore examined the relative contributions of verbal and visuospatial simple and complex working memory to mathematics in primary school children. Children in

Years 2 to 5 (6 to 10 years) were assessed (M age = 100.06 months, SD = 14.47, n=111). Results revealed an age-dependent relationship, with greater visuospatial influence in older children. Further analyses demonstrated that backward word span and backward matrices contributed unique portions of variance of mathematics, regardless of the regression model specified.

A further feasibility study (n = 28) investigated whether the relationships identified were resilient to the inclusion of other cognitive measures and whether there were any underlying cognitive deficits common in poor performers in mathematics. The study explored measuring working memory, speed of processing, g (intelligence), and number sense simultaneously. None of the regression models generated were significant, with no suggestions of fundamental differences between children who performed poorly in mathematics and their peers. Further analysis revealed considerable heterogeneity in the cognitive profiles of children showing a cause for concern in mathematics. The study demonstrated the approach is potentially feasible if the chosen measures thoroughly explore the child's cognitive profile.

A final two-year follow-up to study 1 investigated how subcomponents of working memory measured in Year 3 related to mathematical performance in Year 5 (n = 159 M age = 115.48 months). Results show a shift from spatial-simultaneous to spatial-sequential influence, whilst verbal involvement remained relatively stable.

Possible explanations for the findings in relation to the existing literature are explored along with implications for educators and further research. Consideration is also given to the value of remediation strategies for poor mathematical performance.

The findings of the project as a whole indicate that there is a shift in the influence of the working memory components from spatial-simultaneous to spatial-sequential as children get older. The contributions of verbal working memory remain consistently important at all ages. These results suggest that a screener developed to predict future attainment should include measures of each of these areas in order to account for both shorter and longer term prediction.

Preface

In 2011, the Skills for Life survey found only 22% of people had strong enough maths skills to get a good GCSE in the subject - down from 26% when the survey was carried out in 2003. The Organisation for Economic Co-operation and Development (OECD) school league tables, published 2016, compared 15-year-olds' abilities in core academic subjects across 65 countries and regions in the Programme for International Student Assessment (Pisa). The figures show that the United Kingdom was ranked just 26th for maths, a drop of two places since 2006. Consequently, British schoolchildren continue to lag behind their peers in the Far East despite increases in education spending.

Department for Education statistics indicate that, at the time of proposing this research project, 13% of all pupils in England were leaving primary school having achieved level 3 or below in maths (Department of Education, 2016), with this figure reaching as high as 35% and above in some schools. Understandably, poor achievement at primary school level does not only hinder performance in this phase in isolation; weak maths performance upon leaving primary school then has lasting impacts upon maths achievement at GCSE level (with 7.9% of all pupils in State funded secondary schools falling into the category of "low attainers" based on achieving grades A*-C in English and Maths) and beyond.

I therefore intended to focus on primary aged children, both Key Stage 1 and 2 (i.e. 7 – 11 years). It is believed that the relationship between working memory and maths will be mediated by changes in age related strategy use and the nature of the curriculum (Holmes & Adams, 2006). The project aims to examine the relationship between working memory and maths performance in children aged 7 – 11-years. The work builds upon research identifying a predictive link between working memory and mathematics (e.g. Passolunghi & Cornoldi, 2008; Swanson & Beebe-Frankenberger, 2004; Wiklund-Hörnqvist, Jonsson, Korhonen, Eklöf, & Nyroos, 2016; Wilson & Swanson, 2001). A further aim of the research is to work towards developing a screening assessment to identify children with working memory deficits related to early maths development. An important component of the work is the combination of cutting-edge research with real classroom settings to maximise the ecological validity of the work and make a measurable and sustainable impact in the early identification of children-at-risk of low maths attainment.

This thesis will be presented as a portfolio of individual studies, with sections clearly identifying the progression from one study to the next. As such, the literature review section is dispersed throughout the introduction sections of the individual papers in order that background information is presented where it is most relevant. Papers that have been published have been highlighted using a change of font and information regarding where and when they were published is included in the introductory section of each study.

1

Methodology

This project is located in the literature and the methods of the field of psychology and, more specifically, working memory, with the aim that all of the methods used are presented clearly enough to be replicated (Baltes, Reese, & Nesselroade, 2014; Coe, Waring, Hedges, & Arthur, 2017). My personal background is in Psychology, therefore, these methods are familiar to me and were chosen as they are best placed to answer the research questions that are detailed later in this chapter. I worked as a sole researcher throughout the project, recruiting, collecting, and inputting data alone (Wellington, Bathmaker, Hunt, McCulloch, & Sikes, 2005). I designed all of the studies besides study 2 with the guidance of my supervisors, Professor Steve Higgins and Dr John Adams. For study 2, I collaborated on the study design with Dr David Giofrè, as I did for the analysis of the papers where he is included as a co-author. Being a sole researcher, my personal interests may have influenced the study somewhat for example in the way I designed the studies (Foote & Bartell, 2011). The studies could be considered to support a particular model of working memory, for instance. However, I attempted to mitigate this by avoiding suggesting that the results supported a particular model and providing alternative explanations for results where applicable. I was also able to alleviate some of the issues of personal interest by precisely marking all of the mathematics tests strictly according to their mark scheme, for example a question asking children to identify the three triangles with right angles indicated that children should not receive the mark unless they had correctly identified all three of the target triangles and no others. This process was followed for all mathematics tests administered. Issues of researcher bias or positionality are also arguably mitigated by the publication of the papers as the research developed. Engaging with peer review helps to ensure the validity and rigour of the underpinning research methods and ensures that bias is minimised in the analysis and interpretation of results.

Research questions

Systematic review

1. How does the age of the participants influence the relationship between visuospatial working memory and mathematics?
2. How does the type of maths being assessed influence the relationship between visuospatial working memory and mathematics?
3. How does the type of visuospatial working memory being assessed influence the relationship between visuospatial working memory and mathematics?
4. How does the nature of the mathematics tasks being used (standardised/non-standardised) influence the relationship between visuospatial working memory and mathematics?

Study 1, paper 1

1. How do the subcomponents of working memory relate to the performance of written mathematics?

Study 1, paper 2

1. How does the balance of influence of different working memory subcomponents differ according to the area of mathematics in question?

Study 2

1. Which components of working memory are more influential in mathematics performance at different ages across the primary school years?

Study 3

1. Is the relationship identified between working memory and mathematics sufficiently stable as to remain when additional cognitive control measures are included?

2. Is the approach of assessing children's cognitive capacities to identify those who may fall behind in mathematics in the future feasible?

Study 4

1. Is there evidence for a relationship between working memory measures taken in Year 3 and a mathematics measure taken in Year 5?
2. If so, is this relationship the same as that when the mathematics measure was taken in Year 3?
3. Which working memory predictors are able to predict mathematical performance in Year 5 when mathematical performance in Year 3 is accounted for?

The research questions detailed were developed as the project progressed, beginning with the need to fully understand the literature that was already available in a systematic way, before identifying the gaps in this literature to address through the experimental studies. The overall aim of the project was to understand how working memory relates to mathematics over the span of the primary school years and to establish whether this relationship is stable over time. In each case, the design and research questions for the subsequent study were derived from the results of the previous study to ensure the research was developed from the most up to date findings.

Data Analysis

The data were analysed using structural equation modelling, a form of multiple regression (Kline, 2011), based on the general linear model and using latent variables, that analyses the magnitude of the relationships between the measured variables, using a confirmatory approach. Measured variables can be either observed (in this case mathematics), meaning the construct is measured directly, or latent (in this case working memory), meaning the construct is not directly observable so measures are used that we assume are accessing it.

Latent variables are factor analysed to ensure that those we expect to be measuring the same underlying construct load onto the same factor.

Models in structural equation modelling are defined based on the researcher's understanding of the literature which meant that I was able to specify different models to explore their fit (how well they suited the data) using different assumptions about the structure of working memory. This allowed me to understand how different combinations of components of working memory related to mathematics. I was also able to perform variance partitioning on the data to understand the individual contributions of each component to the prediction of mathematics, rather than only understanding which measures were making a significant contribution, as is the case with standard regression.

The main limit associated with structural equation modelling and variance partitioning is the sample size required to ensure that there is enough data for the number of comparisons being made. This meant that it was not statistically sound to do some more detailed analysis, for example analysing the contributions of working memory components to components of mathematics across different ages in study 2. However, the reliability and validity of the method, and the affordances for understanding the intricacies of the relationship outweigh the limits of the method and support using it to understand the relationship in more detail.

Ethics

Each study involved in this project carried different ethical risks, depending on the design used to answer the research question. I considered at each stage how the different designs led different ethical considerations to arise, as will be discussed below. Individual ethics applications can be found in the appendices (see Appendices A-D), dated 01/12/2017, 31/10/2018, 24/05/2019, 19/08/2019. Throughout, I was working with a vulnerable group (children: Mahon, Glendinning, Clarke, & Craig, 1996; Morrow & Richards, 1996), therefore, there were more ethical considerations to take into account, and a greater number of safeguards to put in place. The process of addressing these considerations was reflexive and adapted to the given situation (Powell, Fitzgerald, Taylor, & Graham, 2012). The design used was correlational and so does not carry as many risks as an intervention study, for example, but perhaps more than a purely observational study as participants are placed in a specific testing situation. To this end, the studies were designed with the ethical codes of the British Psychological Society (BPS; BPS, 2018) and British Education Research Association (BERA; BERA, 2018) in mind. The key points of each of these codes that pertain to this particular project are listed below:

BPS
Respect **Privacy, confidentiality, power, consent, self-determination, compassionate care**
Competence **Necessary skills, limits of competence, advances in evidence base, professional ethics and decision making, caution in making knowledge claims**
Responsibility **Professional accountability, responsible use of knowledge, respect for welfare, potentially competing duties**
Integrity **Honesty, unbiased representation, fairness, avoid exploitation and conflict of interest, maintaining boundaries**
BERA

Responsibilities to participants
Respect, structural inequalities, competence, balance maximising benefits and minimising risk of harm to participants, informed consent/assent, opt-in and out depending on context (e.g. to reduce sampling bias), transparency (open, honest, conflict of interest), right to withdraw, incentives: level shouldn't influence decision to participate, harm (ease of participants, avoid excessive demands, duty of care, rights of individuals, time and effort of long-term research), privacy/data storage (confidentiality, anonymity, waiver of anonymity, secure storage), disclosure
Methods
Analysis techniques, inferences to be drawn from findings
Responsibilities to the community of educational ressearchers
Integrity of the reputation, identify contacts
Responsibilities for publication and dissemination
Communicate findings and practical significance, open access

Throughout my project, I have striven to adhere to these principles. I will discuss below how I managed these ethical considerations with respect to my project under the revised headings of Protection for Participants, Attributes of the Researcher, and Ethical Use of Data for clarity of combining the two ethical codes.

Protection for participants

To tackle the issue of anonymity and confidentiality, children were assigned an anonymous code that was used throughout data collection so that data could not be traced back to each individual. This code was used on all documents surrounding data collection and analysis, and no reference to identifiable individual cases was included in the writing up of the reports so as to maintain anonymity. Another key component of maintaining anonymity is maintaining the trust of the child that their teacher will not find out their results (Einarsdóttir, 2007), however, in one particular case, which will be discussed further in due course, it was to the benefit of the child and their teacher to disclose their results, following obtaining parental permission. Participants did not have access to any data, be that their own or that of other participants, and all data collected is held securely. Should participants wish to have their data removed from the study, they may contact the researcher and their data will be destroyed by the researcher. Only those individuals who require access to the data were granted access (namely, the researchers and supervisor) to either the physical data or the electronic data file. All electronic data is stored on a protected device in order to reduce unwarranted access.

With regard to consent, consent cannot be granted by the child themselves, since they are under 16 (Morrow & Richards, 1996), therefore an alternative process was used. Consent was obtained from the Head Teacher of the school and the class teacher, before a letter detailing the study was sent to parents with an attached opt-out consent form (see appendices E-H for copies). Opt-out consent was used so as not to bias the sample towards more affluent areas with more proactive parents (Krousel-Wood, Muntner, Jannu, Hyre, & Breault, 2006). The use of opt-in consent may have restricted the range of the sample to a lower proportion of disadvantaged children and may have therefore skewed the sample. This was, in fact the case during the follow up study where one school requested opt-in consent be gathered and the participation rate dropped significantly. The overall risk of harm was minimal throughout and opt-out consent also significantly reduced the burden on school staff as fewer administration duties were required before the study commenced, therefore we argued that the ethical concern should be adaptable to the inherent risk (Bromwich & Rid, 2015; Graham, Powell, & Taylor, 2015). Verbal assent was then obtained from each child respectively before beginning the study (Gallagher, Haywood, Jones, & Milne, 2010). The right to withdraw was explained verbally to all pupils at the same time as explaining all other aspects of the study. Children were also given the opportunity to ask any questions they may have had before the study began and at any time throughout. My decision to adopt opt-out consent was partly to ensure compliance, whilst also balancing the needs of ethical research (i.e. obtaining a balanced sample which is more likely to be representative) and ensuring ethical practice (such as minimising the burden on practitioners). It was also balanced by my discussions with the children (ethical engagement, not legally required but ethically important) and the overall potential benefit from the research programme (public good).

The amount of stress placed upon the children involved was taken into consideration. As testing took place in an environment familiar to the children, it was unlikely to lead to increased anxiety levels (Saywitz & Nathanson, 1993). Additionally, the working memory tasks, although novel to most children, did not induce undue stress. The test materials used were taken from standardised test batteries that have been developed with clear and concise manuals detailing the exact format and process for each of the subtests. Children usually enjoy such activities when presented as low stakes games and puzzles in a supportive environment. All subtests were administered in the prescribed way, whilst remaining aware of the child's well-being. If a child had shown any signs of discomfort, they would have been reassured and removed from the situation immediately and appropriate action taken to mediate the situation, such as placing the child in a different classroom with a familiar member of staff. This did not occur at any time during the testing periods for any of the studies in the project, indicating that children were at ease throughout. Children were fully debriefed on the study's purpose in language they could understand and given the opportunity to ask any questions upon completion. Researcher contact details were also provided should any questions have arisen after completion of the study or should a child or parent have wished to withdraw consent. By following the procedures detailed above, the risk of the study was minimal to none for all involved.

A further concern was the time and effort required for long term research from the participants (Barry, 2005; Goodman & Blum, 1996). Children did drop out at various stages of the project, particularly in the final testing phase. This was accounted for in the initial number of participants recruited and was not questioned at any stage. Children had received no incentives to take part and so did so of their own free will. By explaining this to children meant they felt able to drop out of the study if they felt they no longer wanted to participate. They did receive an individual sticker as an acknowledgement of their efforts after each stage, however, no children were notified of this before the study began so as not to influence their decision to take part when giving assent.

Finally, with regard to implications for research participants, at the school level, the main implication for the informants is the benefit of the knowledge obtained through the study. As a school they will then have the opportunity to inform their teaching practice using the results. From the perspective of the teachers, the implications reflect those of the school as a whole. I have returned to each of the schools who took part in the project and delivered a session on working memory in the classroom, incorporating the findings of my project, in order to give teachers the opportunity to develop their own understanding from taking part. Whilst the implications of the research may not be of immediate benefit to the children currently in the classes tested, future children will benefit from the results of the study through the school's use of the findings in the design of their teaching.

Attributes of the researcher

As a researcher entering a school with the expectation that I will be fully informed on everything involved with the project, I made sure I was familiar with the testing procedures for each of the measures I used so as to not waste time or lose data due to errors in administration. Further, I made every effort to remain abreast of the literature that had, and continued to, inform my project to make sure that I could answer any questions the children, teachers, or parents had. This helped put those I was working with at ease and ensure they felt confident to take part in the studies. I was very aware throughout that I was entering the schools with knowledge that was not usually available to children and teachers on an everyday basis. As such, it was important that I made teachers aware that I could not give any kind of diagnosis or make formal recommendations regarding particular children and that I maintained this stance throughout (Lanzi & Ramey, 2013). There was one case, as previously mentioned, where the results gathered from one child were at odds with advice teachers had previously told me they had received for the child. In this case, I thought it was in the child's best interest to disclose my findings in order to inform the teacher of how they might learn best, however, this was not done until explicit consent to do so was received from the child's parent. Even so, results were only then discussed in terms of standardised results, from which raw scores are impossible to derive without a thorough, in-depth knowledge of the data set.

At all points during my data collection, the school, teachers, parents, and children were all fully aware of my purpose for being there and the research was conducted in a professional manner (Lanzi & Ramey, 2013). Children were all treated equally and fairly, with a careful balance in the relationship between myself and the children to elicit the best performance possible from them. The children received encouragement to ensure they put effort into the tasks, but I was mindful that this was not done in a way that would exert power or authority and pressurise any child into taking part. This power imbalance when working with children is addressed in Einarsdóttir (2007).

Ethical use of data

Extensive analysis was performed at each stage of the project in order to obtain as much information regarding the relationships between working memory and mathematics as possible from the data set. It was important to thoroughly explore the data gathered to ensure that participant time was not wasted, especially given data collection had occurred during school hours. At each stage, everyone involved in the project was aware of the purpose for collecting the data and the types of inferences likely to be drawn to ensure transparency in the process at all times. The findings were then used to inform the design of future studies to ensure that only the necessary testing was carried out. Finally, following the analysis of the data, the majority of the resulting papers have been, or will be, published open access in order that they can be accessed by the public, particularly those in education, to inform practice. Alongside this, as previously mentioned, I have conducted feedback sessions with each school in order to inform them of the outcomes of the research, improve teachers' awareness of working memory, and hopefully, inform classroom practice.

Considering all of the above ethical issues in a flexible way, and thus being able to adapt to the ever changing situation of working with schools, has allowed me to gain as much insight as possible from the work I have conducted whilst always maintaining the well-being of those involved in my study as a priority.

Data Management and Data Protection

All data collected for this project were collected and held in accordance with data protection legislation (Chassang, 2017; Dove, 2018) and recommendations (Abbott, 2015; Cooper, 2016). Information was distributed to the schools, teachers, and parents of the children involved regarding the data that would be collected, following approval from the University. Regarding data retention, parents were informed that the information provided by them, and the data gathered from the study, pertaining to their child(ren) would be kept for analysis, though that it would be done so following strict controls. To adhere to privacy regulations, children's data was held anonymously, with the working memory and mathematics data held separately from personal information. All data was also analysed anonymously, with no reference to identifying data included in the data set. Parents were made aware at each stage of the project that they had the right to withdraw their consent for the processing of their child's data (also adhering to the right to erasure). This happened on one occasion, on which the data was destroyed (shredded) immediately. The data in question had not yet been entered into the dataset, so only the physical copy of the results required destroying. Consent was obtained for all children involved in the series of studies by their parent or guardian in order for their data to be included. This data included their full name, gender, and date of birth, provided by the school. Children and parents were allowed to correct their personal data that was held for the study (rectification), however, young children are often unsure of their personal information, such as their date of birth. The mathematics tests used did ask children to indicate whether they were a boy or a girl, therefore, this information could be used to cross reference with the information provided by the school and ensure that information was matched correctly across data sets.

In summary, this thesis is presented as a series of papers representing the development of my thinking throughout the process. The papers are drawn together with a broad introduction to the working memory literature, the mathematics development literature, and the available literature on predicting mathematics from working memory. These more general introductions are included to provide the broader background for the work and to set the project in context, before presenting the relevant, more specific literature for each study in the individual paper introductions. Discussions are presented in each individual paper for the specific study and a general discussion for the project as a whole is presented at the end of the thesis to draw together the findings of the series of studies.

2

Working Memory

A number of definitions have been proposed over the history of working memory research, however, there is an emerging consensus on what working memory encompasses and what its role involves, despite the maintenance of different definitions. Baddeley (1992) defines working memory as the "temporary storage and manipulation of the information necessary for such complex cognitive tasks as language comprehension, learning, and reasoning". This definition appears to be the one that most closely accounts for the elements included in alternative definitions, with the added clarification from Baddeley (1996) that working memory assumes the role of an "interface between perception, attention, memory, and action". It achieves this by way of coordinating the involvement of each of these elements in everyday behaviours. It seems that Baddeley's definition aligns relatively well with the alternative definitions given, with most discrepancies seemingly arising as a result of the use of terminology, as opposed to any fundamental differences in what the theorists describe.

One example of these alternative definitions is that by Randall Engle (2002) who suggests working memory refers to the attention-related aspects of short-term memory. Engle and colleagues suggest working memory is the ability to control attention (as in Engle, Tuholski, Laughlin, & Conway, 1999; Unsworth & Engle, 2007), whereby a higher working memory span is reflective of an increased ability to control attention, and therefore reduce distraction. As a result, this definition is in contrast somewhat to that of Baddeley (1992) which describes a finite store, however, aligns with the notion that working memory encapsulates attention processes. Similarly to Engle, Cowan (2017) states that working memory is "a system of components that holds a limited amount of information temporarily in a heightened state of availability for use in ongoing processing" (in Adams, Nguyen, & Cowan, 2018). Cowan's definition of "generic working memory" is intended to describe the ways in which relatively easier access to information held in mind can help individuals to make use of such information during everyday tasks. Terminology aside, this definition appears to relate to the processing component of Baddeley's (1992) definition. This is by no means an exhaustive account of the numerous definitions given for working memory in the field, however, these definitions align well with the models discussed further in this chapter and highlight some of the overlaps in meaning behind the definitions used, despite a lack of formal consensus on the wording of a definition.

One debate that is prevalent within the literature is that of whether working memory and short-term memory can and should be differentiated. Cowan (2008) suggests that the confusion surrounding the differences between the two is a result, again, of the language used by researchers in their definitions. Some researchers use definitions that lend themselves to there being no clear distinctions between short-term and working memory, for example Miller and Galanter (1960). They studied a more functional memory that allows us to go about our life successfully, by using goals and sub-goals to accomplish tasks and milestones. However, short-term memory is viewed by others distinctly as those tasks that do not require manipulation, instead requiring only verbatim recall (Adams et al., 2018). Daneman and Carpenter (1980) defined working memory tasks as exactly the opposite of these short-term memory tasks; tasks that require simultaneous storage and manipulation. In doing so, they were able to explore how increased processing demands of working memory tasks related to individual differences in storage capacity.

This is not a universal consensus, however, with researchers such as Engle et al. (1999) and Kane, Bleckley, Conway and Engle (2001) arguing the critical distinction surrounds whether tasks are challenging with regard to attentional control. This relates to their definition of working memory, as mentioned above, suggesting that low span individuals will be more distractible as a result of being less able to marshal their attentional resources to focus on the task in hand. Interestingly, this is an important element in understanding why poor working memory manifests as it does in the classroom and one potential reason for the difference in predictive value between working and short-term memory tasks for academic achievement. This point will be discussed further in a later chapter specifically dedicated to predicting academic performance.

Similarly to previous comparisons of definitions, Baddeley (1996) provides a distinction between short-term memory and working memory that can also be applied to these other explanations when the meaning of the wording is considered. He explains that working memory comprises a "number of subsystems, rather than a unitary model" that demonstrates a functional role in task completion, with working memory used as the term for identifying the whole system, rather than just the short-term store (Baddeley & Hitch, 1974). That is to say that short-term memory is a part of working memory, but the functional element is the result of involving these additional processing modules. Following this, it is understandable how working memory as a whole can be seen as a useful place-keeper during everyday tasks, for example during mental arithmetic (Cowan, 2008). This also further highlights how the Baddeley and Hitch (1974) model is able to explain findings predicted by other models.

Models of working memory

As previously mentioned, we are now beginning to see some level of convergence between the many proposed models for working memory, which Adams et al. (2018) suggest should occur as a result of rigorous research practice. Before considering the models that are beginning to converge, it is important to first consider earlier models, such as that of Atkinson and Shiffrin (1968). Atkinson and Shiffrin provided what Baddeley and Hitch (1974) termed the "modal model" as this was the most often cited version of the typical type of model at the time. Their multi-store model has three components: the sensory register, the short-term store, and the long-term store. The short-term store is the component described as the subject's working memory (Atkinson & Shiffrin, 1968). A small amount of information passes from the sensory register into this short-term store where it is combined with long-term memory and held temporarily (Atkinson & Shiffrin, 1968; Shiffrin & Atkinson, 1969). Control processes are then at the disposal of the subject for revisiting information to maintain it in working memory, and for transferring information between working memory and long-term memory (Adams et al., 2018). However, Baddeley (1996) argues that this process relates more to short-term memory than to working memory, with Cowan (2008) adding that it is short-term memory that is responsible for maintaining a small amount of information in a more accessible state for a short period of time. Whilst this model was acceptable at the time to explain short-term memory, it did little to explain some of the phenomena that were being highlighted through research findings. It considered the short-term store to be a unitary concept, which was not readily accepted by later theorists (e.g. Baddeley & Hitch, 1974) due to the apparent presence of different types of storage (i.e. phonological, visual; Cowan, 2017) and findings showing that phenomena such as the recency effect (the increased likelihood for the final items in a list to be recalled) were unaffected by a secondary memory load (Adams et al., 2018). With only a unitary store, this would not be the case since capacity would be reached with only the primary load. Hence, no further information would be maintained. This primary load may be displaced by the secondary load, or the secondary task would not be possible if the requirement was to maintain the primary load.

This discrepancy with the multi-store model led to the development of alternative models. The first that will be discussed here is the embedded process model, developed by Cowan (1988). Cowan (2017) refers to the model as one of "generic working memory" as he does not propose any mechanisms to explain the function of the model, rather focusing solely on information retention. Cowan (1999) defines working memory in relation to this model as "cognitive processes that are maintained in an unusually accessible state", placing the importance on accessibility through attentional processes. The model proposes that a brief sensory store brings in information from the environment, which then activates the relevant areas of long-term memory based on the properties of the incoming information. The new input then overwrites/interferes with the activated information from long-term memory. As with all models of working memory, the information must be regularly refreshed through rehearsal or attention to prevent decay. In this model, dishabituation (a response to a novel stimulus following repeated presentation of a previous constant unchanging stimulus i.e. individuals attention is recaptured again when something in their environment changes; Ropeter & Pauen, 2013) is responsible for filtering the amount of information entering working memory as attention is drawn to changes in the environment. The central executive directs the focus of attention to the relevant information, thus allowing the coherent interpretation of the information, which can then be added to long-term memory to update the previous representation. This acts as an assimilation mechanism for new information with previously held information. There is some evidence that the capacity of working memory in this model is likely to be around four chunks of information (Cowan et al., 2005), however, there are also counter suggestions that the focus of attention can only hold a single item of information at any one time before offloading this to create space for a second set of information (Cowan, 1988, 2001; Saults & Cowan, 2007). The issue of capacity will be discussed further in a subsequent section. The embedded process model lies somewhere between the nomothetic (relating to general scientific laws; Salvatore & Valsiner, 2010) and the idiographic (more data driven in nature, distinct from scientific laws, relating more to the particular; Salvatore & Valsiner, 2010) method in the way it is uses results to inform theory (Adams et al., 2018) in line with the model's suggestion that new information is used to update previously held information.

Multi-component Model (Baddeley & Hitch, 1974)

When considering the available data on working memory, Baddeley and Hitch (1974) argued that performance cannot be explained by a single process. They argued that performance requires the processes of multiple components of the same system to achieve the results they had identified (see Adams et al., 2018 for a summary). It was this explanation of their data that drove Baddeley and Hitch to develop their model of working memory (1974): the multi-component model. This tripartite model (Baddeley, 1996) has been highly cited in working memory literature and has had probably the largest influence on working memory research since it was published. It also appears to be the model that is best able to describe a number of the findings presented by researchers, hence is where we are beginning to see some level of convergence. Baddeley and Hitch (1974) developed a modular theory with information divided based on the type of information in question (e.g. verbal or visual information). Modular theories open up the possibility of explaining findings in terms of one module reaching capacity whilst other information is accounted for by other modules. This allows for the maintenance and manipulation of information over and above that which had been deemed to be the capacity of working memory by other models. This option is not available in instances where the model is not modular (Adams et al., 2018), for example the modal model (Atkinson & Shiffrin, 1968). Importantly, the model was mostly derived from nomothetic inference, however, took some input from idiographic data, especially that of brain damaged patients (e.g. Baddeley, Della Sala, Papagno, & Spinnler, 1997; Robertson, Manly, Andrade, Baddeley, & Yiend, 1997), which allowed Baddeley to understand how the model would function when a specific individual component was removed. In comparison to the embedded process model (Cowan, 1988), in the multicomponent model information from long-term memory feeds into the corresponding component to assist with the interpretation of new information (Baddeley, 2010), as opposed to a small section of long-term memory being held in an active state and using current information to update this section. In the following section, I will discuss the development of our understanding of each of the components of the multi-component model as these elements will be broadly those applied to the following work, with a particular focus on phonological and visuospatial subsystems.

Phonological loop

The module that underwent the most investigation first was the phonological loop, due to the easy accessibility of the subsystem through measures such as digit recall. The phonological loop is responsible for the storage and manipulation of phonological information (Baddeley, Gathercole, & Papagno, 1998) whose usefulness Baddeley argues should not be ignored in research as it has demonstrated widespread use in a number of circumstances (Baddeley, 2011). The phonological loop has been shown to be strongly implicated in developmental milestones including language acquisition, demonstrating that non-word repetition is predictive of language learning for both first and second language learning in children and adults (e.g. Adams, 1996; Gathercole & Baddeley, 1993; Speciale, Ellis, & Bywater, 2004). Importantly, Gathercole (1995) highlighted a bidirectional relationship between the phonological loop and long-term memory, following results showing that non-words with similarities to English are more readily recalled than those that are dissimilar to English. This extends earlier research by Baddeley, Papagno and Vallar (1988) showing a direct link between working memory and long-term memory whereby the phonological loop is implicated in forming phonologically-related long-term memories, particularly those related to language learning. Further research on sign language and lip reading implicated the same system in these methods of communication (Baddeley, 2011; Rönnberg, 2004; Rönnberg, Rudner, & Ingvar, 2004), raising the question of whether the phonological loop functions to serve language more specifically, with other non-language-based sounds being moderated by a different system. Similarly, these results raise questions over whether phonological information must be presented in an auditory manner, given the findings for sign language and lip reading.

To test this, Larsen and Baddeley (2003) presented information visually to participants under articulatory suppression (subjects constantly repeat a simple irrelevant sound such as “the” in order to prevent subvocal rehearsal of the material; Baddeley, 1992) and demonstrated a large cost to the performance of the phonological loop under such conditions. These findings indicate that information need not be presented auditorily in order to access the phonological loop, whilst also highlighting the vulnerability of the loop to external factors, and its reliance on rehearsal. I will discuss the importance of rehearsal further following discussion of the model itself. Again considering articulatory suppression, Baddeley and colleagues have demonstrated that articulatory suppression eliminates the word length effect but not the phonological similarity effect for written words (Baddeley, Thomson, & Buchanan, 1975; Baddeley, Lewis, & Vallar, 1984). The word length effect demonstrates how performance declines as word length increases (namely, the number of syllables in a word; Hulme & Tordoff, 1989). Meanwhile, the similarity effect indicates that sequences containing phonologically similar words are harder to recall than those with phonologically dissimilar words (Conrad & Hull, 1964; Salamé & Baddeley, 1986). That the word length effect is eliminated under articulatory suppression, even when stimuli are presented in a written format, suggests that written material that is phonological in nature is transferred to the phonological domain before it is stored in the phonological loop. Finally, there is the suggestion that “item information may be helped by similarity since it places constraints on possible responses” (Baddeley, 2011). This statement indicates that, although phonological similarity may be detrimental to recall, semantic similarity and repetition from the same restricted set of stimuli can more accurately draw out the ability to remember serial order due to the additional information available. Conversely, using different items for each set is likely to confound studies investigating serial order by bringing in the impact of loss through decay due to the restricted additional information available.

Visuospatial Sketchpad

The second of the slave systems, as reported by Baddeley and Hitch (1974) is the visuospatial sketchpad. This module appears more complex than the phonological loop (Baddeley, 1996), therefore was not as heavily researched immediately after the development of the model. As such, we continue to see a deficit in the volume of research on the visuospatial sketchpad as compared to the phonological loop. The function of the visuospatial sketchpad is defined as the storage and processing of visual and spatial information (RepovŠ & Baddeley, 2006). The most commonly used test of spatial short-term memory is the corsi block tapping task, as developed by Corsi (1972). During the task, the experimenter taps out a sequence on a board displaying nine randomly spaced, fixed blocks (as described in Berch, Krikorian, & Huha, 1998). The subject must then repeat this sequence in the same order. This test is referred to as one of short-term memory as it does not require any active manipulation of the stimuli, rather a direct repetition of the sequence the experimenter taps out. The average span score for this type of task is around five (Monaco, Costa, Caltagirone, & Carlesimo, 2013), showing a lower span than that for digit recall tasks assessing the phonological loop.

Following the corsi block tapping task being considered a measure of spatial span, a measure of visual span was developed, resulting in the visual patterns test. Della Sala, Gray, Baddeley, Allamano and Wilson (1999) argue that this task is tapping an area of working memory that is dissociable from that tapped by the spatial corsi block task. This distinction between visual and spatial has been investigated further to suggest that these elements are separable, yet related. One of the first researchers to demonstrate such a distinction between visual and spatial elements was (Logie, 1986) who determined that some imagery tasks relied on the visual, not spatial, component (see Klauer & Zhao, 2004 for a review of this literature). The opposite had previously been shown by Baddeley and Lieberman (1980). Importantly, neuropsychological research has been conducted with patients showing evidence of double dissociations between the ability to make spatial imagery judgements about objects (e.g. their location in space) and the ability to use visual imagery to make visual judgements (e.g. of shape and colour; Carlesimo, Perri, Turriziani, Tomaiuolo, & Caltagirone, 2001; Luzzatti, Vecchi, Agazzi, Cesa-Bianchi, & Vergani, 1998; Tresch, Sinnamon, & Seamon, 1993). Further, visual involvement was also seen when text processing involved visuospatial information (e.g. imagining yourself moving through a space; Mammarella, Pazzaglia, & Cornoldi, 2008; Pazzaglia, 1999).

Moving away from distinguishing between visual and spatial working memory, another area of considerable interest to researchers is limitations of duration and capacity. Posner and Keele (1967) suggested that visuospatial working memory lasts for only 2 seconds during written letter processing, after which point they suggested visuospatial systems were superseded by verbal systems. This finding suggests that, in terms of the processing of written letters, the immediate code may be visual, but that this is then taken over by a more slowly developing verbal code. Baddeley (2011) suggests a possible influence of the length of the visual trace (the visual stimuli to be remembered) in this situation. Phillips and Baddeley (1971) investigated this claim with non-word visual stimuli, requiring subjects to make same/different judgements of matrices. They found a decline in performance as a function of the length of time the visual trace was required to be held, similar to that of Kroll, Parks, Parkinson, Bieber and Johnson (1970), suggesting that the result identified by Posner and Keele (1967) is more likely to result from participants switching from a verbal to visual code due to the use of letter as stimuli.

Logie (1995, 2001) suggests that rehearsal may occur in the visuospatial sketchpad to increase retention time, drawing a distinction between visual and spatial information. He suggests that visual information is likely rehearsed in the visual cache, whereas spatial information is likely to be rehearsed in the inner scribe. This rehearsal mechanism can be used to explain how information can be maintained for longer than the estimated duration of the visuospatial sketchpad. Finally, Phillips (1974) highlights the capacity limits of the visuospatial sketchpad, showing a decline in performance as a function of the number of cells in the matrix used as a stimulus. That being, as the matrix increases in size (e.g. from 4 x 4 to 5 x 5), the accuracy of recall declines, giving evidence that the visuospatial sketchpad is a capacity limited module. Both temporal and capacity limits will be discussed further in due course.

Central Executive

The central executive is the component of the multi-component model where we begin to see the influence of attention. Baddeley (2011) states that he views the central executive as a homunculus; rather than suggesting the central executive as an explanation for phenomena, he describes it as an identification of areas requiring further research in order to understand them properly. To this end, Baddeley (1996) suggested four possible functions of the central executive: to focus attention, to divide attention, task switching, and to act as a long-term memory interface. In cases where patients appear to have lost the functions of the central executive due to neuropsychological damage, the term dysexecutive syndrome is often used (Baddeley et al., 1997; Baddeley & Wilson, 1988). The naming of these deficits as a syndrome suggests that the loss of function of the central executive results in a collection of symptoms which regularly occur together, thus making it difficult to ascertain the individual functions of the central executive in isolation. Siegel and Ryan (1989) have shown support for the notion that the central executive exerts control over the other systems, particularly in the form of attentional control, with Adams et al. (2018) adding that this control appears to be strategic.

They suggest one example of this strategic control is the initiation of rehearsal strategies to prevent the loss of information through decay.

Tasks assumed to load onto the central executive are generally those that require attention to complete, for example counting backwards in 7s from 352 (Baddeley, 2011). This is in sharp contrast to the basic repetition tasks used to assess some other components of working memory. Is it reasonable in this case to distinguish between short-term memory and working memory tasks by suggesting working memory tasks require an executive component? One commonly used measure of the central executive is the reading span task whereby subjects are required to make a semantic true or false judgement about a sentence as well as retain the last word of the sentence. Once all true/false judgements have been made, subjects are required to recall the last words in the order they were heard (Daneman & Carpenter, 1980). This task is used to predict comprehension skills by predicting an individual's capacity to draw inferences and extrapolate information among other skills demonstrating a definite executive component. Cain, Oakhill and Bryant (2004) found that measures of working memory thought to access the central executive predicted unique variance of reading comprehension in children, in line with the assumption that central executive functions are likely related to more demanding cognitive tasks.

The central executive is also implicated in concurrent task completion, supporting Baddeley's speculation of its role in dividing attention. For example, deficits are seen on both tasks when digit span and visual tracking are completed simultaneously, especially when done by patients with Alzheimer's disease (MacPherson, Sala, Logie, & Wilcock, 2007) who are expected to show executive functioning deficits. This deficit remains even after task difficulty is titrated to show equivalent performance across groups on individual tasks. A finding of this nature suggests a dissociable component, thought to be a result of the central executive, highly affected by Alzheimer's disease, which deteriorates more rapidly than in healthy ageing (MacPherson et al., 2007; Morris, 1994).

With regard to the other proposed functions of the central executive, one such is that it acts as an interface with long-term memory. Cowan (2008) suggests that it serves to reduce working memory load by grouping the required information into a smaller number of units. This suggestion is in line with that of Siegel and Ryan (1989) that working memory holds the stimulus information whilst retrieving other information from long-term storage. It is plausible that it is this interaction with long-term memory that is responsible for the findings of Baddeley, Vallar and Wilson (1987) showing an interaction between phonological and semantic coding with memory for semantically related words reaching spans of around 15 words, yet unrelated words reaching spans of around 5 words.

Episodic Buffer

The episodic buffer is the most recent addition to the multi-component model, added as a cross modal memory store to explain the links formed between pieces of information, for example names and faces, or other pairings of semantic information held in memory (Baddeley, 2000). In a review of working memory research in 2011, Baddeley clarified that the episodic buffer was included in an attempt to demonstrate the "fractionation of the central executive into separate attentional and storage systems". It is clear that the purpose of the episodic buffer is to hold semantic information (Adams et al., 2018), however, Cowan (2008) suggests that it may be the episodic buffer that relates more to the focus of attention than do other components of the model. As such, the episodic buffer is not specialized for any particular kind of information, but may show some stronger experimental links to attentionally driven tasks (Adams et al., 2018). Its links to long-term memory are explained by the suggestion that the episodic buffer is "assumed to hold integrated episodes or chunks in a multi-dimensional code"

(Baddeley, 2011), thus becoming the link between perception, working memory, and long-term memory.

As with all components of working memory, the episodic buffer is also limited in capacity, demonstrating a capacity for roughly four chunks of information (Cowan, 2005). It was assumed initially to be dependent on the resources of the central executive as the processes it was involved in seemed to be attentionally demanding (Baddeley, 2011). Feature binding studies have sought to understand these capacity limits more intricately and have revealed a capacity of around four objects, regardless of the features required to be remembered (e.g. Vogel, Woodman, & Luck, 2001). This suggests the episodic buffer does not distinguish between types of information, rather between groups (chunks) of meaningful information. A binding deficit only becomes evident when an item other than the final item in the stimulus set is probed during recall, suggesting it is the maintenance of bound features against distractions that requires additional attentional resources (Allen, Baddeley, & Hitch, 2006). Baddeley, Hitch and Allen (2009) found similar results for verbal bindings, demonstrating that concurrent attentionally demanding tasks do not interfere with the binding process itself, rather with the maintenance of the bound features. The conclusion, therefore, is that the episodic buffer is an "important but essentially passive structure on which bindings achieved elsewhere can be displayed" (Baddeley, 2011), whilst also allowing for further manipulation after the initial binding phase, for example binding phrases into sentences and objects into scenes.

Capacity Limitations

The most well-known figure given for the capacity of working memory is Miller's magic number seven (1956). In this number, Miller states that the capacity for working memory is seven items plus or minus two; that is capacity should range from five to nine items. However, this figure was derived from measures of short-term memory that do not necessarily reflect the ability to hold unrelated information in mind, not least perform manipulations on the information. Cowan (2008) suggests that the figure might be relatively accurate in adults, though suggests that not all items are stored as a separate entity. Many researchers have claimed that the limit is more likely around three or four units of information when strategies to overcome capacity limits are accounted for (Broadbent, 1975; Cowan, 2001; LeCompte, 1999; Warfield, 1988). Further claims have also been made regarding visual working memory, estimating its capacity at three to five items in young adults (Cowan, 2010), consistent with verbal working memory estimates. Baddeley (1996) states that capacity has been reached when the “first item has faded from memory before the last item has been processed”, suggesting that typical memory capacity is likely to be around six to seven digits because serial order is lost beyond this point (Baddeley, 2011). He places an emphasis on the loss of serial order as defining the upper limit of working memory because many tasks in everyday life are reliant upon serial order, including language or the necessary steps for executing a skilled action such as kicking a ball (Baddeley, 2011). Without the ability to retain these chunks of information in a serial order, all meaning would be lost.

Semantics and attention also seem to play a part in capacity, as capacity for lists of semantically dissimilar words may be reduced because of the lack of detail captured for complex items (Adams et al., 2018). Semantically similar lists also provide a smaller number of possibilities subjects can recall from. With regard to attention, word lists that were ignored at the time of presentation, rather than immediately recalled, are typically recalled with a span of 4 plus or minus one item (Cowan, 2001). Cowan suggests this may be more reflective of capacity as no strategies such as rehearsal or chunking are being deployed. A large amount of inter-individual variation is evident in capacity, varying from two to six items in adults, but fewer in children (Cowan, 2008; Gathercole, Pickering, Ambridge, & Wearing, 2004), with capacity limit seeming to correlate with cognitive aptitude (Cowan et al., 2005). Baddeley et al. (1975) and Baddeley (1986) proposed that working memory capacity is dictated by the number of items that can be read or recited in a two second window, indicating that capacity may also be bound by time constraints of how many items can be recalled before decay begins to take effect. Barrouillet, Bernardin and Camos (2004) agreed in principle with this notion, however suggested that refreshing of information could be done via attention and was not necessarily dependent on rehearsal, as had previously been suggested by Baddeley. They proposed that high cognitive load/ distractor tasks engage attention elsewhere and prevent refreshing. A combination of these two explanations could be used in conjunction to explain more recent results (e.g. Camos, Mora, & Oberauer, 2011; Vergauwe, Barrouillet, & Camos, 2010) who showed that adults prefer to use attentional refreshing when stimulus materials contain phonologically similar items, relying on rehearsal when a concurrent task is attentionally demanding, but that verbal and visuospatial recall is reduced when completing a concurrent task that is high in cognitive load. We also see some evidence of intra-individual variation, for example in individuals with developmental disorders (e.g. Alloway, Gathercole, Kirkwood, & Elliott, 2009). This study compared the working memory profiles of individuals with different developmental disorders in order to highlight strengths and weaknesses. It demonstrated that those with language impairments demonstrated deficits in verbal working memory, whereas those with motor impairments showed deficits in visuospatial working memory. The remaining areas of working memory functioned as expected in each case. This indicates the possibility that a single individual may have a relatively uneven working memory profile, though this is not always to a clinical level associated with a developmental disorder, as seen in Alloway et al. (2009). We see a similar pattern with regard to amnesic patients who show preserved capacities for some elements of working memory, yet severe deficits in other areas as a result of brain lesions (e.g. Baddeley & Wilson, 1988, 2002).

One area particularly pertinent to this project is the development of working memory throughout childhood. A plethora of studies have investigated this and have demonstrated that children as young as four years of age have a working memory profile that fits relatively well with the multi-component model (e.g. Gathercole et al., 2004), with its systems seemingly separable across childhood (Jarvis & Gathercole, 2003; Pickering, Gathercole, & Peaker, 1998). Pickering et al. (1998) evidenced this by showing that the phonological loop and visuospatial sketchpad are independent structures in children aged five and eight years old. Gathercole and Pickering (2000) then went on to determine that the central executive and phonological loop are separate but associated in six and seven-year-old children, which is also in line with the adult model. Findings of this nature suggest that development may be responsible for increasing the capacity of each of the individual components, rather than undergoing a fundamental change in structure. It is, however, suggested that the visuospatial sketchpad is not dissociable from the central executive in six and seven-year-old children, indicating that there may be some element of structural change before children reach adulthood (Gathercole & Pickering, 2000), particularly relating to the higher-order executive components of working memory. Verbal and visuospatial components appear to be independent of each other in 11 and 14-year-olds (Jarvis & Gathercole, 2003), consistent with findings demonstrating this in younger children, with Gathercole et al. (2004) suggesting that structure and capacity appear to be adult-like by the mid-teenage years.

There is some discrepancy around the age at which we begin to see these increases in capacity with the earliest proposition being that this begins at 4 years of age (Gathercole et al., 2004) in the phonological loop, central executive, and visuospatial sketchpad. Children's visuospatial (counting arrays of coloured spots) and verbal (memory for a series of tones) working memory was demonstrated to improve with age (Cowan, Li, Glass, & Scott Saults, 2018). However, Adams et al. (2018) questioned whether these improvements were the result of increased capacity, or due to improvements in processing speed (allowing quicker refreshing). This is not a question that is easily answerable without conducting studies using methods to prevent rehearsal and refreshing, but these kinds of tasks are very difficult to do with children due to the additional task demands of concurrent tasks or articulatory suppression. Many other studies have demonstrated similar findings for working memory capacity increase in sentence span and counting span (Siegel & Ryan, 1989), change detection (Riggs, McTaggart, Simpson, & Freeman, 2006), and visual patterns task and block tapping (Logie & Pearson, 1997), while some studies have attempted to distinguish capacity growth from other cognitive factors. Cowan (2016) and Cowan et al. (2018) suggested that results indicated that working memory capacity was increasing across development even after accounting for other cognitive factors such as distractibility and rehearsal.

Paying particular regard to processing speed, memory decay is known to have a large influence on children's memory span (Towse, Hitch, & Hutton, 1998). Children with a faster processing speed generally have higher working memory spans, suggesting that processing time is an important limiting factor for the amount of information children are able to hold in their working memory. Case, Kurland and Goldberg (1982) demonstrated this by showing a linear relationship between processing speed and memory capacity on a counting span task. These results suggest that older children are faster at the processing portion of the task and, hence, were able to hold more information in their working memory. One reason for this is that stored items may be lost if the processing phase takes longer, because stored information begins to decay before it is refreshed (Towse & Hitch, 1995). Interestingly, in the Case et al. (1982) study, adults completing a counting span-type task using nonsense words reflected the performance of six year olds on the counting span task. It was suggested that the familiarity of the non-words was relatable to the six-year-olds familiarity with counting words and may highlight a trade-off between processing and storage because more memory was devoted to remembering an unfamiliar sequence. When processing was faster, more resources were available for storage and hence participants scored higher on the task (Towse et al., 1998). It is reasonable then to suggest that we may not be seeing such a vast increase in working memory capacity as originally thought, and rather an improvement in processing speed, leading to less forgetting. This is the conclusion drawn by Towse and Hitch (1995) who found that "for each age group, span was a function of the total duration of the counting operations. Complexity of counting had no effect beyond that attributable to count duration". That is to say that storage and processing are independent of each other, and children complete these tasks by switching between processing and storage. They argued it is unlikely that a result of this nature is due to shared resources due to the lack of influence of counting complexity. These results highlight the importance of using scaled scores in order to make direct comparisons between different age groups on working memory measures as the findings taken together suggest continuous development and improvement in capacity from four or five years of age to adulthood.

One potential way to counter the capacity limits of working memory is to employ chunking as a strategy. This strategy groups information into a smaller number of units of to-be-remembered information. Broadbent (1975) suggested that chunks typically comprise three items, for example when participants were asked to recall states in the US, they typically recalled them in groups of three. However, when investigating the number of chunks recalled, a number of studies propose that this may increase (e.g. Ottem, Lian, & Karlsen, 2007) due to strategy development (e.g. Case et al., 1982; Towse, Hitch, & Skeates, 1999), and language ability (e.g. Ottem et al., 2007). One possible mechanism for this finding in terms of language ability is an increase in the ability to form associations between items. Gilchrist, Cowan, & Naveh-Benjamin (2009) investigated this over development and demonstrated that the number of chunks recalled increased as a function of age, however found no evidence that the amount of information contained in each chunk changed. In a developmental population, Pascual-Leone (1970) has previously identified that the number of chunks remembered by children increased with age as older children were able to remember more stimulus-instruction bindings, suggesting this finding is relatively robust. Running span tasks make chunking difficult as the participant is unaware of when the list will end, therefore recall appears to be limited to three to four items (Bunting, Cowan, & Scott Saults, 2006), as explained previously regarding working memory capacity. However, Tulving and Patkau (1962) suggested that it is possible to facilitate chunking, in which case subjects were able to recall four to six chunks of information. They also identified that the number of items per chunk in their study was related to how similar the non-words used were to English words, thus suggesting that familiarity and semantics may influence the size of chunks formed. Additionally, free recall and long lists seemed to facilitate chunked recall, as in Chen and Cowan (2005), who demonstrated that six pairs were remembered as well as six individual items. Early work by Glanzer and Razel (1974) seems consistent with more recent work that three to four chunks is the likely limit, although the size of each of the chunks is still to be understood.

Temporal restrictions

As mentioned previously, working memory is very short lived and cannot be brought back once lost through decay. Though the exact time limit is disputed, all estimates are in the order of seconds. Baddeley (1996), referring to the phonological loop, suggested that decay occurs in two to three seconds without rehearsal, whereas Engle (2002) proposed that non-rehearsed material will be lost in around 20 seconds. Keppel and Underwood (1962) demonstrated that three items could be recalled equally well after different delay periods when no previous lists had been presented, thus suggesting working memory may last for longer than first thought. However, this finding is ungeneralizable to real life as human beings very rarely encounter truly novel situations where no previous experience is available to cause interference.

In order to try to negate the effects of time, it is possible to mentally rehearse stimuli to revive the memory trace (Baddeley, 1996), effectively resetting the clock on memory decay.

Rehearsal as a practice becomes less effortful with age (Guttentag, 1984), but as a process is poorly conceptualized (Baddeley, 1996). Baddeley suggests it becomes less effortful with age because it relies on introspection, which is beyond the capabilities of young children. Rehearsal is usually done subvocally, though covert rehearsal can achieve the same maintenance effect, particularly in visuospatial working memory paradigms (Godijn & Theeuwes, 2012). In line with his model of working memory, Cowan (1992) suggested that mentally attending to the stimulus materials could reactivate the memory sufficiently well as to function as a form of rehearsal. Support for this idea was found when Barrouillet et al. (2004) and Barrouillet, Bernardin, Portrat, Vergauwe, & Camos (2007) demonstrated that including an attentionally demanding task between the to-be-remembered items reduced recall. One explanation for this is that attention is "used up" by the distractor tasks, leaving insufficient amounts of a limited resource to attend to the to-be-remembered information. A similar method used to prevent rehearsal is articulatory suppression (Alloway, Kerr, & Langheinrich, 2010; Baddeley et al., 1975) which has been shown to impair tasks in the verbal domain.

A body of research also exists surrounding the use of visual rehearsal, suggesting that it is possible to detect eye saccades when subjects are using visual rehearsal strategies (Tremblay, Saint-Aubin, & Jalbert, 2006), indicating that the mechanism for this type of rehearsal might be similar to that of "retracing your steps", which may in turn be different to using visual imagery. Tremblay et al. (2006) highlighted this as a successful form of rehearsal, showing that greater overt visual rehearsal leads to better serial recall over and above the effects of rehearsal based on shifting spatial attention. This is concluded because preventing overt rehearsal lowered performance to levels equal to when no rehearsal took place. Visual rehearsal is shown to induce activation in the contralateral early visual processing areas relative to the presentation of the stimulus (Awh et al., 1999), highlighting one potential mechanism by which rehearsal in the visual domain may operate.

As previously mentioned, visual imagery can also be used to overcome the temporal limitations of working memory, however, is less practiced than rehearsal so relies on the central executive to a greater extent because of this (Baddeley, 1996). Visual imagery has been shown to improve retention for the purpose of comprehension of texts (Chan, Cole, & Morris, 1990), however, like other rehearsal strategies, is not flawless. Visual methods were impaired when subjects completed a concurrent spatial task, thus suggesting that the same underlying component is depended upon for each task (Gyselinck, De Beni, Pazzaglia, Meneghetti, & Mondoloni, 2006). As these results suggest, interference can be seen for visual techniques resulting from a distractor in the same domain, for example visual noise can be seen to disrupt visual maintenance (Quinn & McConnell, 2006).

3

Mathematics

Pre-school development of mathematics

Development of number sense

Despite the intuitive expectation that mathematics education begins once children enter formal schooling, a significant part of a child's basic understanding of number and mathematics develops before they reach school age (Mcintosh, Reys & Reys, 1992). This development is termed "number sense". Butterworth (1999) proposed that "number is an innately specified module" in human cognition, indicating that humans have an inborn capacity to represent, process and understand number (Dehaene, 2001). This supposition is supported by Dehaene (2001) who defines number sense as "our ability to quickly understand, approximate, and manipulate numerical quantities". This definition emphasises the rapid nature of the ability, suggesting that, particularly at this age, the ability in question is a non-symbolic representation (Bonny & Lourenco, 2013). Number sense is particularly relevant to humans' capacity to discriminate between sets of different sizes (Giaquinto, 2018), making comparisons of their size/magnitude relative to each other (Mcintosh et al., 1992). As expected, number sense develops with experience (Mcintosh et al., 1992).

Dehaene et al. (1999) suggests two systems of representation: approximate and exact. They suggest that approximate representations are analogue and independent of language, with the variability in the signal elicited proportional to the magnitude of the set size. Conversely, exact representations are both language and culture dependent. Dehaene et al. (1999) posit that exact representations of numbers are discrete and represented using integers. Each of these representations explains different findings from the literature, strongly suggesting that both have roles in development (Ansari & Karmiloff-Smith, 2002; Carey, 2001; Dehaene, 2011) and providing evidence that numbers can be represented in different ways, and that these different ways may be more or less helpful or appropriate depending on the situation (Mcintosh et al., 1992). An extension of this model of representation is the triple code model, proposed by Dehaene and Cohen (1995), proposing that numerical information can be represented in three ways: analogue (as described previously), verbal (numbers as strings of words), and visual Arabic (digits). Dehaene (2001) goes on to explain that language, culture, and teaching are responsible for developing the lexicon, written notation, and formal procedures for completing formal maths.

The latter is an intuitive claim since it is these formal operations children begin to learn when they enter formal schooling, however, the lexicon, and often written notation, begin to develop before formal schooling begins. Hence, mathematical learning does not begin at the point of school entry; it is evident long before this point. Analogue, verbal, and visual Arabic representations are often seen as modular representations in young children, thus explaining many of the errors young children make when learning mathematics, such as misreading, incorrect comprehension, transformations, incorrect processes, and inaccurate encoding (Watson, 1980). However, as mathematical cognition begins to develop and improve, these representations become less modular and so fewer of these types of errors are made. Fewer modular errors are the result of the emergence of transcoding. This means information can be translated directly from one form to another, and develops as children begin to develop their understanding of number. However, new procedures developed through instruction must cohere with the child's current understandings of mathematics (Resnick, 1984). As a result, children are able to understand mathematics, rather than rely solely on fact retrieval and repetition. Although, some elements of mathematics do rely more heavily on fact retrieval than others, for example multiplication tables (De Smedt & Boets, 2010; Imbo & Vandierendonck, 2007). Nevertheless, we can observe the use of transcoding in these situations as calculations are based on a set of input and output codes, and often recited as verbal associations between strings of words, however, can also be recognised in written form. Addition facts can be checked for whether the proposed answer is likely to be correct by judging the distance between the operands and the answer given (Dehaene, 2001), however, this is not as easy to do for operations such as multiplication. Hence the increased reliance on memorisation. Dehaene (2001) suggests that close approximations in multiplication are often more confusing because the answers for neighbouring calculations are also activated in response to the calculation in question (see Baroody, 1992 for an explanation).

Considering the comparison of numerosity, object file theory proposes that each object is stored in a different file, hence sets can be compared based on their quantity by way of one-to-one correspondence between the number of "full" drawers (Carey, 2001; Simon, 1997; Uller et al., 1999). However, one of the major constraints of this theory is that the magnitudes that can be compared are limited by the number of available files. Some researchers argue that discrimination of numerosity follows Fechner's law (Dehaene, 2001), a generalisation of Weber's law, stating that as quantities get larger, a greater difference between the two quantities to be discerned is necessary for their accurate distinction. The time taken to compare numbers has also been proposed to take longer when the magnitude effect increases (this is the smaller number of the two to be compared – the larger the smaller number of the two, the longer the comparison will take) and the distance effect decreases (this is the distance between the two numbers to be compared – the closer the two numbers are in the number sequence, the longer the comparison will take; Giaquinto, 2018). Giaquinto (2018) highlights that magnitude comparison is also more error prone in these situations. However, this is countered by Inglis and Gilmore (2014) who argue that explaining the acuity of the approximate number system using Weber fractions and numerical rations demonstrates poor test-re-test reliability. They also indicate that numerical ratios are not related to measures of Weber fractions or accuracy, and that Weber fractions produce skewed data sets. Studies of this nature suggest that researchers making assumptions on the approximate number system based on these measures should exercise caution in the conclusions they draw.

Gallistel (1990) proposed that number sense developed evolutionarily to track food sources, predators, and mates. The sense was necessary to make comparisons to determine where is “more”. One major difference between these proposed origins and current research around the approximate number system is the inclusion of numerals. Learning symbols for words is never found in the wild, this aspect always requires training, suggesting that exact symbolic processing is not used by animals, but is not beyond their capabilities with very specific training (Dehaene, 2001). On the other hand, non-symbolic representations are readily available, hence these skills developing before formal schooling begins (Mcintosh et al., 1992). Number sense is argued to be a single entity that develops from infancy to adulthood (Lipton & Spelke, 2003) as evidence shows that ability to successfully discriminate between numerosities requires a smaller ratio in 9-month olds (a ratio of 1.5) than in 6-month olds (a ratio of 2). As previously mentioned, this evidence should be interpreted with caution, given that ratios are the quoted measure, however, this is the measure used in the vast number of studies of this kind. Early work with human infants demonstrated the ability to distinguish exact numbers for sets up to set size three (Antell & Keating, 1983; Starkey & Cooper, 1980; Strauss & Curtis, 1981), as well as the ability to discriminate between quantities of two and three (Ansari & Karmiloff-Smith, 2002; Starkey & Cooper, 1980; Strauss & Curtis, 1981). This evidence of the early ability to make distinctions between numerosities does, however, appear to be limited to set sizes up to three. Feigenson & Carey (2005) showed infants are not able to extend the same principles to differentiate between one and four, suggesting infants may lack the cognitive access to cardinal numbers above three (Giaquinto, 2018).

The fact that infants fail discrimination tasks at quantities above three suggests they are subitizing, rather than applying analogue processes to do such tasks (Ansari & Karmiloff-Smith, 2002). Assuming infants are subitizing during these tasks also relates well to the effect of the distance between the numbers being compared (Dehaene, 2001). In defining subitizing, Dehaene and Cohen (1994) state it is "the ability to rapidly name the numerosity of a set of simultaneously presented objects when it is below three or four, but not beyond". That is that it is a visual process (Mandler & Shebo, 1982; Trick & Pylyshyn, 1994) possible only for the set sizes for which infants can successfully pass the task. Since this is a visual process, one may therefore expect that subitizing may relate to visuospatial working memory, particularly in early mathematics learning. Studies have investigated the stability of subitizing ability in primary school children, showing an ability to identify whether a display contains one, two, or three items using this method (Benoit, Lehalle, & Jouen, 2004). Le Corre and Carey (2007) found children aged 3-5 years old could state how many items were in a set up to set size four when cards were presented too quickly for the dots to be counted. This finding suggests a development in subitizing ability since infancy, with accuracy on this task indicating that these children have enduring representations of these set sizes (Giaquinto, 2018). With regard to the developmental trajectory of subitizing, adults demonstrate the ability to make successful quantitative judgements for quantities up to four (Ansari & Karmiloff-Smith, 2002), indicating its stability beyond early childhood.

Beyond the ability to subitize with very small set sizes, infants are able to make successful judgements between four, eight, and 16, showing that they possess the ability to represent quantities greater than four (Brannon, 2002), though through a different mechanism. It can be argued that this provides evidence for the emergence of analogue representations in later infancy. Further evidence for the development of quantitative judgements is brought by Wynn (1992) who demonstrated the ability of five-month olds to track basic manipulations of addition and subtraction. Infants of this age showed surprise when they observed a different number to the expected total of objects behind a screen when they have watched them being placed there. Wynn suggests this provides evidence that the infants are able to compute basic arithmetic as they know what to expect having watched the manipulation take place, however, this could also be due to their ability to keep track via object-file theory (Dehaene, 2001). Evidence against an alternative explanation to object-file theory shows infants are able to distinguish between eight and 16, given that object-file theory would provide insufficient capacity for such a task (Xu & Spelke, 2000). Further, evidence in support of hypotheses that infants are responding to number, rather than total area and other alternative explanations, comes from Kobayashi, Hiraki, Mugitani and Hasegawa (2004) and Kobayashi, Hiraki and Hasegawa (2005). They found that children looked for longer when the number of toys dropped behind an opaque screen was unequal to the number of collision sounds they heard when the rate and total duration of the sounds was controlled for. Giaquinto (2018) argues that the results seen are not the result of recognising because the representations tested are too short term, and must be enduring to be considered true recognising. This being the case, the children must be forming new representations each time in order to make comparisons.

With regard to making comparisons in older, more experienced individuals, numerical benchmarks are often used to provide "mental referents for thinking about numbers" (McIntosh et al., 1992). Generally, those such as midpoints, multiples of 10 or 20 are used to judge number magnitude, as are rounded numbers in order to make calculations easier to process. Understanding there is an "orderliness" to numbers, another skill that develops later as children become more familiar with numbers, which often manifests itself in patterns, for example orders of 10 (McIntosh et al., 1992), is also useful for completing calculations. Humans tend to use numerical distance to make estimates for calculations (Ashcraft & Stazyk, 1981). Having in mind an approximate answer to a given calculation can then be used to help with the identification and rejection of obviously incorrect results. The number size effect can, however, interfere with both accurate and approximate calculations, meaning larger numbers often lead to slower and more error prone calculations than smaller numbers (Dehaene, 2001). That being said, human adults are quicker at distinguishing between digits with greater distance between them than those closer together, including two-digit numerals. Dehaene, Dupoux and Mehler (1990), suggesting that the evidence we see in infants and animals still holds in adults, even though they also possess the ability to complete complex problems with accuracy. That these effects are still seen in adults is suggestive of a common underlying mechanism used by both infants and adults of converting symbolic representations of number into their analogic forms before completing comparison tasks (Dehaene, 2001). This mechanism appears then to be pervasive across development, but seems more strongly influential in formal mathematics in children who are weaker in mathematics (Bonny & Lourenco, 2013). McIntosh, Reys and Reys (1992) argue that inefficient strategy use is an indicator of poor number sense because it demonstrates a lack of awareness of the different representations available for calculations, as well as the different methods available. For example, those with poor number sense often revert to remedial counting strategies, rather than use approximation, which is usually borne of practice and familiarity. It is important to note, however, that approximate number system measures on different tasks in adults were unrelated (Gilmore, Attridge, & Inglis, 2011), suggesting that it may not be a singular, unitary system responsible for number sense over the lifespan.

Counting principles

The counting principles are five, arguably implicit, principles that children must understand in order to be considered able to count and understand the count sequence (Gelman & Gallistel, 1986). These principles have been shown to be highly influential in children's future development in mathematics so warrant discussion here. They are:

1. Stable order
2. One to one correspondence
3. Cardinality
4. Abstraction
5. Order irrelevance

This section will address each of these principles in turn in order to understand what each refers to.

Stable order

Stable order refers to the understanding that the numbers in the count sequence must remain in the same order and cannot be substituted. LeFevre et al (2006) argue that the stable order principle is in place by kindergarten, suggesting very early development, most probably due to the early exposure of children to the count sequence. Gelman and Meck (1983) previously argued that three and four year olds recognise the use of the number sequence, being able to identify instances of incorrect order, however, it was counter-argued that simply producing a string of number words in a particular order does not mean that the children understand sufficiently to be classified as grasping the principle (Fuson & Hall, 1983). Children of three and four years of age also struggled to identify cases where a number had been skipped, indicating a less than thorough understanding of the correct number sequence. Further evidence was also provided for counting in a stable order, even if the count sequence was unconventional (Baroody & Price, 1983), meaning that evidence of stable order does not always indicate a true understanding of what the count sequence should be.

One to one correspondence

One to one correspondence refers to the acknowledgement that each item in the array corresponds to one number in the count sequence. In this way, each item should be counted once and only once. Wynn (1992) explains that evidence from young children implies this principle is in place by two to three years of age, despite previous arguments that it is not evident children understand this principle until the age of four and five years (Briars & Siegler, 1984). Wagner and Walters (1982) argue that children do not exhibit adherence to this principle until later than Gelman and Gallistel (1986) claim, rather they have a tendency to re-count items in the array until their count sequence has been exhausted. Baroody and Price (1983) found no evidence that this was the case, however, only examined the counting of four preschool children. A sample of this size cannot be used as sound evidence against the argument of Wagner and Walters (1982), however, provides scope for further investigation regarding the age at which this principle is successfully and consistently applied by children learning to count. Importantly in this debate, children of three and four years of age are able to accurately detect when a puppet double counts or skips an object in an array (Gelman & Meck, 1983), indicating that these children have understood that each item should be counted once and only once.

Cardinality

This is the understanding that the final number in the count sequence used when counting items in an array represents the total set size (Bermejo, 1996). Children are able to count sets of items months earlier than they become aware that the final number in the count sequence represents the quantity (Fluck, 1995), but do not recognise the association between these two concepts; an association that is influenced by mothers' interaction with their children when asking children to 'count' and 'how many' (Fluck, 1995). Approximately 50% of the preschool children tested by Freeman, Antonucci and Lewis (2000) demonstrated the ability to recognise a correct count or a miscount, showing an emerging understanding of the relationship in this age group, however, that this is by no means universal. Their finding is in line with that of Wynn (1990) who argued that only children older than three and a half years understood the cardinality principle. The findings of Freeman, Antonucci and Lewis (2000) also support those of Gelman and Meck (1983) who demonstrated that three to four year olds recognised the error in a count when the number given as the total was not the same as the final number in the count, however, stretched our understanding further by suggesting children are able to also identify a miscount if objects have been missed or double counted. The existing literature is relatively consistent in suggesting children begin to develop an understanding of cardinality around three to four years of age.

Abstraction

Abstraction concerns the acknowledgement that not only physical objects can be counted. Children are thought to understand the principle of abstraction when they are able to count things that are not tangible to them, such as time, planets, distance, etc. Wynn (1990) suggests that children have begun to understand the abstraction principle by around two to three years of age as they are able to count sounds and actions, as well as objects. Children of this age were most successful when counting objects, but most were able to count the sounds and actions accurately. However, Starkey, Spelke and Gelman (1990) argue that infants understand the abstraction principle much before they are able to speak, demonstrating an awareness of numerosity in different modalities. One potential explanation for why such young infants are able to do this is proposed by Greeno, Riley and Gelman (1984) who define abstraction as an "absence of restriction" as opposed to a constraint. In such a way, children may not be able to articulate what they are doing, but still be unhindered by restrictions placed on the to-be-counted set.

Order irrelevance

This principle refers to the understanding that items in a set can be counted in any order without changing the cardinality and is described as the principle that "distinguishes counting from labelling" (Gelman & Meck, 1983). Whilst Gelman and Meck (1983) demonstrated that children aged three to four years old do not object to objects being counted in a random order, later studies have shown that most five to 11 year olds believe order is important (Kamawar et al., 2010). In this study, children showed preferences for left-right or top-bottom counting. Only a small number of 10-11 year olds counted in an order-irrelevant way, suggesting that this principle may not be fundamental to one's ability to count (Kamawar et al., 2010). Baroody (1993) argues that children do not show evidence of understanding that counting objects in a different order produces the same answer, instead arguing that children are able to understand that they can assign tags to objects in any order before they understand that this does not alter the cardinality of the set (Baroody, 1984). Hence, assigning tags can be seen as a less developmentally advanced task than understanding the implications for set size (Baroody, 1984). To add to this, Cowan, Dowker, Christakis and Bailey (1996) demonstrated that children were more likely to state that a recount would produce the same total when they did not have to say what that total number would be. In doing so, they suggest that children may be less confident in their own counting abilities but are aware that the set size will not have changed. Findings such as these suggest there may be different levels of understanding of this principle.

With regard to the application of these principles in everyday life, Wynn (1990) argues that children learn the words for, and meanings of, smaller numbers first. This is a logical claim since children are more likely to be exposed to these words first when taught to count in order, beginning with "one". It is reasonable to suggest that stable order, one to one correspondence, and cardinality underlie early counting as order irrelevance and abstraction do not directly have a negative influence on a child's ability to count. However, Butterworth (2005) suggests a more structured acquisition, with stable order developing first, followed by one to one correspondence, and then cardinality. Evidence for this comes from children's ability to count a set of objects before they are able to answer the question "how many?" (Frye, Braisby, Lowe, Maroudas, & Nicholls, 1989). Understandably, children's counting is often superior to their ability to detect principle violation errors (Briars & Siegler, 1984), providing evidence for the argument that counting develops initially, followed by the understanding of the principles. In support of this, Briars and Siegler (1984) demonstrated that younger children were less likely to object to unusual counting orders (indicating some level of understanding of the order irrelevance principle), as well as one to one correspondence and stable order violations (conversely indicating a lack of understanding of these principles). On the other hand, Stock, Desoete and Roeyers (2009) found that more children had developed an understanding of the one to one correspondence principle than the stable order and cardinality principles by the end of kindergarten. Some level of caution must be applied when interpreting these results with regard to rate of development of understanding due to the differences in administration of the measures. Some studies require children to count themselves, whereas others require children to moderate the counting of a puppet or the experimenter. To this end, there may be an influence of both confidence in their own ability, or misunderstanding the purpose of the questioning techniques, for example thinking they have been asked to recount because they miscounted initially. The effect of set size should also be considered. For example, Gelman and Meck (1983) identified no effect of set size on children's ability to apply the counting principles, but the children were not required to do the counting themselves, thus reducing the task demands. The effect of set size was, however, present when children counted the objects for themselves. The ability of children to apply the principles to the counting of a puppet suggests that they grasp the principles before they fully master the ability to count confidently themselves.

Development of mathematics in school

Entering school signifies the beginning of learning formal calculations to build on the principles children have learned beforehand. Numeracy is defined as "a high degree of ability to cope with current mathematical demands on the community" (Crowther, 1959, in Mcintosh et al., 1992). In order to develop this competence, children are taught according to a spiral curriculum, first introduced by Bruner (1960) with the hypothesis that "any subject can be taught in some intellectually honest form to any child at any stage of development". To this end, younger children are taught the foundational principles first and these are then followed by the more complex aspects later, building on prior knowledge. A spiral curriculum aims to break down the divides seen between subjects in the curriculum (Harden, 1999) and build knowledge through "an iterative revisiting of topics, subjects or themes throughout the course". Each time a topic is revisited, subject knowledge is extended, and currently understood principles are built upon to ensure further understanding is built on solid foundations. Bruner (1960) states that "the end state of this process [is] eventual mastery of the connexity and structure of a large body of knowledge", meaning that a curriculum designed in this way should lead children to reach a level of understanding that can be described as mastery of the subject, as opposed to relying on recall. A spiral curriculum provides value through reinforcement, moving from simple to complex, using integration, logical sequences, higher level objectives, and flexibility (Harden, 1999).

Schmidt, Houang and Cogan (2002) suggested that a few pre-requisite topics were covered in the early years, with more complex topics covered in the later years. This reflects the spiral curriculum leading to positive outcomes as this pattern was identified in countries where children performed well in maths. Snider (2004) argues that the spiral curriculum disadvantages children from low social economic backgrounds, however, this paper was written around schools in the United States and so does not directly compare to schools in the UK, but does still provide an important point for consideration. As an alternative, following the argument that the spiral curriculum encourages topics to be taught one year then forgotten about before being returned to the following year, Snider (2004) suggests a strand curriculum in which topics are addressed for a long period of time and in a great deal of depth until mastery is reached. Snider suggests that a strand curriculum is less likely to discourage integration across topics than a spiral curriculum. Another disadvantage of the spiral curriculum is highlighted by Gibbs (2014) who highlights an important issue that teachers engage with the curriculum in different ways and may not necessarily pick up where the child's previous teacher left off. Hence, the spiral may not always function effectively. He suggests the way around this is team/collective planning within teaching communities or schools to identify exactly where children will have reached in their understanding before they are picked up by their subsequent teacher. He also suggests that the spiral should also occur within each year itself to encourage the development of academic skills and the use of processes of increasing complexity. One suggestion of how this can be achieved is through the amount of scaffolding in place during the instruction phase, with this scaffolding being removed as the year goes on and children develop competence. Importantly, in any curriculum design, there is a need for coherence (Knight, 2001), which should be achievable easily in a spiral curriculum as it should be evident how topics build on one another. The design of the curriculum is a critical factor in student achievement in mathematics (Crawford & Snider, 2000), in which information should be linked directly back to the previous time it was visited both to reinforce what was learned previously and to create links between the information (Dowding, 1993).

Primary Mathematics National Curriculum in the UK

The National Curriculum in the UK is divided into four key stages, however, here I will focus on Key Stages One and Two as these span primary school. The aims of the National Curriculum are as follows:

1. "Become fluent in the fundamentals of mathematics, including varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately". This first aim shows a clear link to the idea of a spiral curriculum,

building on previous knowledge to develop understanding, however, one important aspect of this to be mindful of is that children who have not achieved proficiency in one stage will not be able to build upon their knowledge effectively at a later stage (Department for Education, 2013). This is akin to building a house without solid foundations; the structure of the resulting building will not be sound.

2. "Reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language". This aim requires sound mathematical understanding at each stage to achieve as it relies heavily on abstract concepts. Further, "following a line of enquiry" is likely to load heavily on working memory so may prove incredibly difficult for those children with very poor working memory capacity.
3. "Can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions". In order to do this, children must have a thorough understanding of abstract concepts to be able to apply them to novel situations. Breaking tasks down into simpler steps does suit those with poor working memory, however, not necessarily so if they must form these smaller steps themselves.

These aims highlight how interleaved the topics taught on the National Curriculum are. They are separated necessarily for teaching purposes, but children should be in a position to "make rich connections" across ideas once they have understood each topic. There is a certain degree of flexibility for when each topic is introduced, as long as they are introduced by the end of the relevant key stage, which allows teachers to determine how they believe their children will learn best. This also provides support for Gibbs' (2014) suggestion of collective planning within a working spiral curriculum. The use of practical resources is encouraged in the curriculum, for example concrete aids such as blocks and counters, however, there is an emphasis placed on practice, especially with regard to fluency. A further clause in the National Curriculum states that "pupils should read and spell mathematical vocabulary, at a level consistent with their increasing word reading and spelling knowledge". This may disadvantage children with specific difficulties, especially those who show discrepancies between non-verbal and written ability as it seems to presume that all children progress equally in all areas. This is not the case for those with specific difficulties, for example reading difficulties, who may be able to complete more advanced tasks than their reading ability allows them to access independently. This is something teachers should be mindful of when giving children assistance to complete tasks.

The National Curriculum is divided into topic areas, and further subdivided into statutory and non-statutory elements. Statutory requirements must be met by all children, however non-statutory requirements and specific examples need not be met or used word for word. It is the statutory requirements that will be discussed for the remainder of this chapter. With regard to teaching methods, very few are prescribed. These are the "formal" methods for the four operations: column method for addition and subtraction, short multiplication and division, and long multiplication and division.

Number

The number portion of the curriculum begins in Year 1 with establishing fluency in counting to 100, with some counting in basic numerals, however, this is revisited more thoroughly in Year 2. The characteristics of a spiral curriculum can be seen throughout this development of number fluency from Year 1 to Year 6, by which time the topics included have expanded to include numbers and place value (introduced in Year 2), counting in multiples e.g. of 4, 8, 25, 1000 (Years 3 and 4), negative numbers (Year 4), and rounding (Year 4). By Year 5, children are also expected to understand Roman numerals to 1000, hence requiring a thorough understanding of how the number system works, and by Year 6 to handle numbers to 10,000,000. Year 1 children are expected to develop an understanding of basic addition and subtraction to 20, which again develops over the primary school years to include rule of addition and subtraction (Year 2), using mental methods of calculation (Year 3), and written addition and subtraction with more than four digits by Year 5. Multiplication and division, and fractions/decimals/percentages progress in a similar way from Year 1 to Year 6, with each year building on the previous and increasing the volume of content, however, space must also be made for ratio and algebra by the time children are in Year 6. By this age, the number element of the curriculum alone has over 20 individual elements that children are required to understand, many of which draw heavily on working memory resources which may prove challenging for a number of children, particularly when coupled with the increasing depth of knowledge required at each stage. Whilst number is not alone in its increasing volume of material children are expected to master before leaving primary school, it does show the vastest increase.

Measurement

Measurement follows a similar structure to number, with children in Year 1 being introduced to a relatively small number of practical problems, money, and time, before these areas of enquiry are expanded through to Year 6. Perhaps one of the clearest overlaps in curriculum areas is evident in measurement as it concerns the perimeter, area, and volume of shapes. When working on these components of the curriculum, beginning in Year 3 with perimeter, overlap with geometry should be evident to children. Those who have fully understood previous work on geometry should, theoretically, be in a position to use this knowledge to inform their current learning. However, it can prove difficult to encourage children to make abstract links such as these so often additional help is required. Also included in measurement is the concept of time, which is arguably the most abstract topic to teach to children, with no real clear concrete aids available. Children are, nonetheless, introduced to time in Year 1, and expected to tell the time in Year 2, before being required to convert units of time and solve time-based problems in Years 3, 4, and 5. Teaching such an abstract concept with children as young as 5 years of age poses the risk that they will not be able to grasp the basics, meaning any future work will not be building on sound foundations, as is the aim of any spiral curriculum.

Geometry

Geometry is introduced in much the same way as number and measurement, with the basic naming of shapes and directional commands in the first instance. The curriculum then circles back to discuss properties of these shapes in increasing levels of complexity in the following years. By Year 6, children are expected to be able to draw 2D shapes, describe 3D shapes, and compare and classify geometric shapes. This progression in expectations is rapid, particularly in relation to understanding the more abstract nature of 3D shapes that cannot be easily visualised by some. Children are introduced to the properties of shapes in Year 3, where they are expected to be able to recognise angles and parallel and perpendicular lines. This is, again, revisited to expand their knowledge of angles and regular/irregular polygons in subsequent years. The final element of geometry covered over a number of years on the National Curriculum is grid co-ordinates, and transitions and transformations, beginning with descriptions and later moving on to carrying out transformations and plotting shapes.

Statistics

The fourth and final element of the Primary Mathematics National Curriculum is statistics. This element is not introduced until Year 2 and contains the fewest topics throughout. Statistics begins with constructing and interpreting basic graphs and charts, moves on to presenting and interpreting different types of data (discrete, continuous) from graphs and charts, and has a final expectation that children will be able to construct and interpret pie charts and line graphs by Year 6. Concurrently, children are to be taught to answer basic statistical questions, initially through counting. In the style of a spiral curriculum, these questions and methods develop, upon revisiting the topic, into multistep questions including sum, difference, and means as an average.

It is important to consider, however, that, whilst considered for the purposes of the curriculum as distinct areas of skill, they are not, in fact mutually exclusive, and basic numeracy skills are required for each (as discussed in Holmes & Adams, 2006). Therefore, children's basic ability in this area is likely to be influential in all other areas of mathematics.

4

Using Working Memory to Predict Mathematics

Our current understanding is that working memory is a distinct cognitive correlate of academic achievement, that accounts for a larger proportion of the variance of academic performance than IQ (Alloway & Alloway, 2010). Working memory correlates most strongly with measures of intellect and cognitive aptitude (Cowan, 2008), acting as a mediator for IQ in predicting mathematics in first grade (Passolunghi, Mammarella, & Altoè, 2008). Passolunghi et al. (2008), importantly, demonstrated that working memory predicts mathematics achievement longitudinally when performance IQ does not. Ashcraft & Krause (2007) extend our understanding of this relationship further by suggesting working memory is critical for mathematics performance for any activity beyond simple memory retrieval. This relationship between working memory and mathematics appears to remain consistent across the lifespan, with no difference in magnitude between adults and children (Wilson & Swanson, 2001), hence indicating its use as a long-term predictive measure of later attainment.

Working memory tasks cover a number of different abilities from basic recall (though purists argue that this ability is a reflection of short-term, rather than working, memory ability; Cowan, 2008), recall following a manipulation of the information, and recall of information from a primary task under the load of a concurrent secondary task (dual task). These complex span tasks, performed with the additional memory load are associated with mathematics performance at seven and 14 years of age (Gathercole, Pickering, Knight, & Stegmann, 2004) when mathematics is assessed using National Curriculum assessments. Van den Bos, van der Ven, Kroesbergen, & van Luit (2013) highlighted the importance of noting the mathematics measure used since even though all components of working memory are associated with mathematics, the strength of the relationship can be "explained by the type of mathematics measure used". They argued that more general measures of mathematics yielded stronger correlations, perhaps due to the larger variety of questions drawing more heavily on each working memory component. The contributions of verbal and visuospatial working memory components to mathematics are distinguishable (Giofrè, Donolato, & Mammarella, 2018a), with each showing unique links to mathematics at Key Stage 2 and 3 (Jarvis & Gathercole, 2003).

Involvement of working memory components in mathematics

Results revealing the differential involvement of working memory components in mathematics are mixed. There are a handful of studies suggesting a greater influence of verbal working memory, however, the majority are in support of a larger proportion of visuospatial working memory. Wilson & Swanson (2001) demonstrated that verbal working memory predicted a greater proportion of the variance of mathematics when entered before age, suggesting that the proportional influence of each of the components may alter depending on age. This finding was, however, the result of using arithmetic and mathematical computation tasks, which could demonstrate a greater reliance on verbal working memory if heavily word-based. Especially since children with arithmetic difficulties have lower phonological span scores than their typically developing peers (Hitch & McAuley, 1991; Siegel & Ryan, 1989; see McLean & Hitch, 1999 for an opposing argument). Supporting the suggestion of age-related differences, De Smedt et al. (2009) identified the unique influence of the phonological loop in second grade mathematics, whereas first grade was predicted uniquely by visuospatial working memory.

Visuospatial working memory and measures of central executive have been consistently linked to mathematics performance in young children (e.g. Gathercole & Pickering, 2000; St Clair-Thompson & Gathercole, 2006), with children with arithmetic deficits and/or developmental dyscalculia showing deficits on tasks of visuospatial working memory (Fletcher, 1985; Mammarella, Hill, Devine, Caviola, & Szucs, 2015). With regard to measures of visuospatial working memory, poor mathematics performance is related to poor performance on simultaneous and sequential measures, with those with mathematical learning difficulties the worst affected, followed by those of low mathematical ability, then typically developing children (Mammarella, Caviola, Giofrè, & Szűcs, 2018). Ashkenazi, Rosenberg-Lee, Metcalfe, Swigart, & Menon (2013) demonstrated similar findings of a specific deficit in visuospatial working memory in children with mathematics difficulties when compared to typically developing children.

Using working memory to predict academic attainment has gained a significant amount of traction over the past two decades, with the majority of studies highlighting the strong predictive nature of visuospatial working memory in mathematical attainment. An early study by Holmes & Adams (2006) investigated the relationships between working memory measures and mathematical performance in children in year three and year five in the UK (ages 7-8 and 9-10). Their findings demonstrated that visuospatial working memory was able to predict a greater proportion of unique variance in younger children when mathematics was broken down into "performance-related factors". However, importantly, even when mathematics was broken down according to the key areas of the National Curriculum, the phonological loop was not predicting unique variance, only the visuospatial sketchpad measures were. These findings are supported by a number of further papers presenting visuospatial working memory as a unique predictor of mathematics (e.g. Giofrè, Donolato, & Mammarella, 2018b), particularly in younger children (e.g. Bull, Espy, & Wiebe, 2008; De Smedt et al., 2009).

Subsequently, visuospatial working memory has been subdivided into visual and spatial measures, suggesting children who are poor at solving problems in mathematics fail specifically on spatial, but not visual or verbal, tasks (Passolunghi & Mammarella, 2010). This was supported in 2012 by further findings by Passolunghi & Mammarella highlighting the same pattern of failures in children with mathematics learning difficulties. This pattern does not, however, appear limited to those with mathematical difficulties, as mathematics is also predicted by spatial working memory, but not visual, at the beginning of first grade (Fanari, Meloni, & Massidda, 2019). Although these results did highlight an age-dependent change in influence, with domain general predictors becoming the only significant predictors by the end of second grade. This indicates that visual and spatial working memory are equally influential at the end of second grade, however, the study does not take account of verbal factors, focusing only on the subcomponents of the visuospatial component.

Domain generality and domain specificity

Domain generality and specificity has been a consistent debate in research into the influence of working memory on mathematics. Since the investigation into the specific relationships between working memory and mathematics continues, there is no resolution to the specificity/ generality debate as yet. Domain specificity suggests that the influential working memory components will be dependent on the domain or subdomain of academic performance in question, whereas, the domain general approach suggests no such specific relationships, particularly with relation to academic subdomains. Working memory is often seen as a domain general vulnerability (as in Ashkenazi et al., 2013) as it does not vary depending on whether the subject researchers are assessing its relationships with is reading or mathematics. This explanation aligns well with some other models of working memory, for example that by Engle (2002) regarding the role of working memory in the control of attention. However, there is an increasing body of literature examining the specificity of the relationships between academic performance and its components. Recent research concerning the unique contributions of working memory components and subcomponents to various domains and subdomains of academic performance seems to indicate a leaning toward the importance of specific interactions. This is potentially due to prediction studies and the specificity of the multicomponent model of working memory (Baddeley & Hitch, 1974). Particularly, studying children with specific deficit profiles, allows researchers to examine the specificity of the associated working memory impairments in learning difficulties (e.g. Siegel & Ryan, 1989).

Engle, Cantor, & Carullo (1992) suggest domain generality is evidenced when verbal and visuospatial components both contribute to a higher order performance task as it is drawing on a common resource that is not split into different modalities. For example, we see high levels of association between the visuospatial sketchpad and the phonological loop in complex span measures of visuospatial working memory (Gathercole, Pickering, Ambridge, & Wearing, 2004), which potentially stems from the function of the central executive. In line with this idea, Wilson & Swanson (2001) found some unique variance of task performance was accounted for by the central executive measures that was not otherwise explained by verbal and visuospatial measures. They claim that mathematics drawing on both verbal and visuospatial working can be justified as evidence for domain generality because they are drawing on executive skills that are not from a specific modality. Interestingly, Swanson & Sachse-Lee (2001) examined subgroups of individuals with and without reading difficulties and identified no significant differences in recall of different types of information when groups were matched for executive function abilities. This pattern was echoed by comparisons of groups with high and low executive function abilities showing greater or worse recall on measures, respectively. These differences also remained after controlling for phonological ability, with no significant differences in working memory performance between the groups. These findings are suggestive of a mechanism that operates entirely independently of those required for task completion, in which case, there is a very definite scope for a domain general influence, which may act as a controlling mechanism over more domain-specific measures of the working memory measures.

Further, De Smedt et al. (2009) identified a similar pattern of relationships between the central executive and mathematics in first and second grade. They found that measures of the central executive were predictive of unique variance of first and second grade mathematics when working memory was measured at the beginning of first grade. From this finding, it is reasonable to suggest that the involvement of the central executive in mathematics tasks shows some longitudinal stability. However, with regard to predicting mathematics attainment, it is important to determine whether this relationship exists only for the central executive or for the other working memory components as well. To examine this, a number of studies have been carried out with increasing specificity to identify the cognitive correlates of mathematics performance, with a number of these studies highlighting specific deficits in working memory relating to poor mathematics. One such example is the 2013 paper by Ashkenazi et al. who identified a specific deficit in visuospatial working memory in children with mathematics difficulties compared to typically developing children. They did, however, justify this deficit as a domain general deficit because they argued the deficit had an equal influence on all areas of mathematics. This is unclear, though, and leaves scope for further research in this thesis as it seems logical that mathematics questions assessing spatial skill and understanding would draw more heavily on visuospatial working memory than those assessing numerical manipulations and word problems. In line with this assumption, there is evidence available that children with arithmetic difficulties showed deficits on non-verbal tasks only (Fletcher, 1985). This is indicative of an underlying deficit in the visuospatial domain.

A number of other studies have also been conducted that indicate a specific deficit in particular components of working memory. Siegel & Ryan (1989) showed that children with reading difficulties and arithmetic difficulties showed different working memory profile to each other and from typically developing children. In addition, different mathematics tasks have been shown to be hindered by tasks targeted to occupy components of working memory intended to interfere in its capacity to assist in the completion of tasks (Lee & Kang, 2002). They identified that verbal dual tasks interfered with the completion of multiplication, whereas, subtraction was hindered by a spatial dual task, suggesting the differential influence of components depending on the area of mathematics being assessed. However, this was not found when the tasks were matched for set size and difficulty. The influence of working memory domain may then be reduced when task demands are accounted for (Cavdaroglu & Knops, 2017). We must be cautious though that it is not likely possible to account for task demands when assessing working memory contributions in relation to National Curriculum assessment, and as such, there may be greater demands on components of working memory depending on the mathematical component being assessed by each question. Therefore, we should be conscious of the demands these measures are placing on working memory components.

Other known links between cognitive measures and mathematics performance

Number Sense

Number sense (Greeno, 1991) refers to "elementary intuitions about quantity, including rapid and accurate perceptions of small numerosities and the ability to compare numerical magnitudes, to count, and to comprehend simple arithmetic operations" (Berch, 2005, pp. 334), hence is a logical potential predictor of mathematics attainment. It is an imprecise, nonverbal system that supports basic computation (Feigenson, Libertus, & Halberda, 2013). Jordan, Kaplan, Locuniak, & Ramineni (2007) examined this relationship and identified that number sense at the beginning of first grade is highly correlated with mathematics performance at the end of first grade. They also identified that growth in number sense was predictive of better performance in mathematics. Number sense is a potential predictor of mathematics that can be explored as early as in preverbal infants. Starr, Libertus, & Brannon (2013) took advantage of these early measurement opportunities and examined preverbal number sense at six months old. They identified that number sense ability at 6 months old predicted formal mathematics performance at three and a half years old, thus raising the question does sound early number sense predispose an individual to developing better mathematics skills? This has been demonstrated to be the case on a number of occasions (e.g. Feigenson et al., 2013), however, despite being a "strong longitudinal correlate of arithmetic skills", by the age of six, only knowledge of numerals predicted arithmetic growth (Göbel, Watson, Lervåg, & Hulme, 2014). One suggestion for why this is the case is that the strength of the approximate number system is less important than a good grounding in number knowledge. The prediction of mathematics from number sense shows further flaws when used to predict National Curriculum test scores, with approximate number system acuity measured in preschool being unable to predict non-numerical mathematics ability at age six (Mazzocco, Feigenson, & Halberda, 2011). Again, this suggests the limited potential use of number sense as a long-term predictor, which is perhaps better suited to predicting a child's ability to develop sound number knowledge, which is in turn predictive of their future mathematical ability. However, it is important to note that children with mathematical learning disability were identifiable using a number sense measure (Geary, Bailey, & Hoard, 2009), but that this measure was not able to predict response bias. On balance, number sense has been shown to be predictive of mathematics in young children, however, does not demonstrate capacity to predict over extended periods of time.

Speed of Processing

Processing speed refers to "the speed with which children and adults execute basic cognitive processes" (Kail & Ferrer, 2007, pp. 1760) and as such should be considered as potentially important in relation to mathematical performance. A number of studies have found an association between speed of processing and mathematical performance, including when other factors have also been taken into account. For example, Vanbinst, Ghesquière, & De Smedt (2015) identified that speed of processing did not mediate the association between numerical processing and arithmetic, whilst Swanson & Kim (2007) demonstrated that speed of processing is independent of working memory in predicting mathematics performance, but still correlates significantly. Similarly, processing speed has been shown to account for a unique portion of the variance of mathematical performance on Standardised Achievement Test (SATs) measures when general cognitive ability is accounted for (Rohde & Thompson, 2007), as has it been shown to be the best predictor of mathematics after controlling for reading ability (Rebecca Bull & Johnston, 1997). Here the literature implies that the speed at which children are able to process information is highly influential in their learning. Taken together, these findings indicate the potential for using this measure as a long-term predictor of mathematics performance, however, since it does not mediate the involvement of working memory, it should not be used alone; there is potential to use speed of processing to predict mathematics in conjunction with working memory once this relationship is understood fully.

General Intelligence (g)

General intelligence is often included in studies as a logical cognitive correlate of achievement, but also to investigate whether it mediates the relationship between the chosen cognitive correlate and the academic component being tested (as in Kyttälä & Lehto, 2008). If g mediates the relationship between the two, it is more likely that the initially observed relationship was actually the result of general intelligence, rather than the specific cognitive correlate examined. G factor has been demonstrated to correlate significantly with mathematics (as would be expected since mathematical ability is a component of a standard IQ test; Campos, Almeida, Ferreira, Martinez, & Ramalho, 2013), as well as predict growth in mathematical competency (Chu, vanMarle, & Geary, 2016), hence is an important measure to consider here. Similarly, variance in working memory has been shown to correlate with g once the shared variance has been factored out (Engle, 2002; Engle et al., 1999), though this was not the case for short-term memory variance. Examining its specific relationship with mathematics, Kyttälä & Lehto (2008) found a predictive relationship between fluid intelligence and mathematics, where g predicted unique variance in mathematics and mediated the influence of active visuospatial working memory. Hornung, Schiltz, Brunner, & Martin (2014) also demonstrated that intelligence predicts arithmetic and number line estimation once early number competencies have been controlled for. However, they identified that this relationship was not present for shape-based skills. This suggests that shape-based mathematics relies on a different cognitive correlate than g.

Given these findings, there is a need to understand the relationships identified further to understand how they relate to mathematics over the primary school years and to ascertain whether these relationships remain stable over time. Only once we have a thorough understanding of these relationships longitudinally will we be in a position to make effective use of the knowledge for the early identification of children who are likely to struggle.

5

Introduction to the Systematic Review

Before conducting any additional research on the relationship between working memory and mathematics, it is first important to understand the existing literature, and what this means for the current project, in a meaningful way. There is a growing body of literature available that investigates the working memory-mathematics link (Alloway & Alloway, 2010; Rebecca Bull et al., 2008; Giofrè & Mammarella, 2014), including distinguishing between verbal and visuospatial working memory. This differentiation has been made by a number of researchers, for example by Mammarella et al. (2006) and Passolunghi & Mammarella (2010), who demonstrated evidence for subdivisions within the working memory domain. However, there is very little thus far that examines this link with respect to the differential contributions of the types of visuospatial working memory: simultaneous and sequential. It is possible to subdivide visuospatial working memory into simultaneous and sequential tasks easily, depending on the presentation of information, given these types of tasks allow for such differences in presentation format. When presented simultaneously, all visual information is made available at the same time for the given duration, before disappearing to allow for the recall phase of the task. In contrast, sequential tasks present the visual information in series, meaning each element of the stimulus set is presented one at a time before disappearing to make way for the next. Participants are instructed to recall the sequence in whichever way was requested upon the completion of the presentation of the sequence. Differentiating between simultaneous and sequential presentation would be much more difficult with verbal (spoken) information, as verbal information is, by default, sequential since only one word can be said at any one time. If the definition of verbal information is extended to include written information, it is possible to present the information simultaneously much more easily, however, this introduces additional potential confounds, such as reading ability. The following systematic review seeks to understand the current literature on the relationship between mathematics and working memory, including the subdivision of visuospatial working memory and mathematics. The following paper is published in Educational Psychology Review (Allen, K., Higgins, S., & Adams, J. (2019). The relationship between visuospatial working memory and mathematical performance in school-aged children: A systematic review. *Educational Psychology Review*. doi:10.1007/s10648-019-09470-8; Appendix I). It was authored by myself with guidance from Prof. S. Higgins and Dr. J. Adams.

6

The relationship between visuospatial working memory and mathematical performance in school-aged children: A systematic review

Abstract

The body of research surrounding the relationship between visuospatial working memory and mathematics performance remains in its infancy. However, it is an area generating increasing interest as the performance of school leavers comes under constant scrutiny. In order to develop a coherent understanding of the literature to date, all available literature reporting on the relationship between visuospatial working memory and mathematics performance was included in a systematic, thematic analysis of effect sizes. Results show a significant influence of the use of a standardised mathematics measure, however, no influence of the type of visuospatial working memory or mathematics being assessed, on the effect sizes generated. Crucially, the overall effect size is positive, demonstrating a positive association between visuospatial working memory and mathematics performance. The greatest implications of the review are on researchers investigating the relationship between visuospatial working memory and mathematics performance. The review also highlights as yet under-researched areas with scope for future research.

Introduction

Development of children's mathematical skills

Informal mathematical development begins much before children reach the age of formal education with the development of number sense. Inherent in this is the existence of a mental number line (Berch, 2005; Schneider, Grabner, & Paetsch, 2009). As a precursor to the development of this mental number line, research postulates an innate sense of number by which humans are able to distinguish between sets to judge which has more (the approximate number system; Dehaene, 2001). Additionally, young children demonstrate the ability to perceptually determine the exact number of items in small sets (Clements, 1999); an ability known as subitizing (Benoit et al., 2004; Ginsburg, 1978). This innate sense of number is a skill that evolutionary psychologists attribute survival to, for example where one can find more food (De Cruz, 2006). Many habituation studies (e.g. Starkey, Spelke, & Gelman, 1990; Xu & Spelke, 2000), have provided evidence for number sense in young infants, demonstrating a renewed interest upon alteration of the number of items in the presented array, as long as a critical ratio criterion is met (according to Weber's Law; Feigenson, Dehaene, & Spelke, 2004).

Once children become verbal, they learn a counting list which functions in the form of a "placeholder structure" (Sarnecka & Wright, 2013), carrying little numerical context. This suggests that children develop a knowledge of a specific set of number words, in a fixed order, before their knowledge develops into a deeper understanding of number as an abstract principle (Sarnecka & Gelman, 2004). A further milestone in the development of number sense occurs when young children are taught to attribute specific quantities to Arabic numerals (Krajewski & Schneider, 2009a). Wynn (1990) previously described specificity as the knowledge that every number word describes a specific numerosity. Importantly, the attribution of specific quantities to individual numerals paves the way for children establishing understanding of a set of rules: cardinality (the final numeral used represents the total number in the set), abstraction (sets of any nature can be counted, including entirely mental constructs), one-to-one correspondence (each item in a set should be counted once and only once), stable order (numerals should be used in a fixed order), and order irrelevance (items in a set can be counted in any order without changing the cardinality of the set; Baroody, 1984; Dehaene, 1992; Thompson, 2010). Upon reaching this stage children are deemed to have developed a “mental number line”, which, over time, becomes increasingly linear after initially following a somewhat logarithmic structure (Stanislas Dehaene, 2003; Siegler & Booth, 2004), whereby numbers outside of the child’s counting range may be viewed only as “big” or “lots”. From this foundation, children can begin to understand the formal manipulations of numbers required to gain proficiency in mathematics through formal instruction, as identified by Libertus, Feigenson and Halberda (2011) who demonstrated that ANS acuity in infants predicts early maths achievement.

The development of mathematical skills, upon the commencement of formal schooling, can be considered to pertain to two broad stylistic categories, as adopted by Weschler assessments: Numerical Operations and Mathematical Reasoning. Whilst the National Curriculum has four areas (number, measurement, geometry, and statistics) it is these categories that will be considered in this review as they succinctly describe the fundamental understanding of mathematics (Numerical Operations) and its application (Mathematical Reasoning). Numerical Operations concerns procedures that may best be described as numeracy, involving number knowledge, basic numerical manipulations, and mental arithmetic (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). Tests of Numerical Operations typically comprise of explicit mathematical equations with basic operations for children to solve using a written format, as well as assessments of counting, identifying numbers, and written calculations (Pearson Clinical; Wechsler, 2017). By contrast, Mathematical Reasoning is defined by Thompson (1996) as the ability to carry out “purposeful inference, deduction, induction, and association in the areas of quantity and structure”. Such a definition aligns well with the nature of the tasks used to assess the construct, which comprise mainly of single- and multi-step contextual story problems that the children are required to solve using the information provided. Examples of such problems are those involving whole numbers, fractions and decimals, graphs, and probability (Wechsler, 2017).

A broad range of assessments are employed in both research and educational settings when establishing a child's understanding of mathematics. Such assessments range from simple, individually derived series of calculations and equations to subtests of standardised test batteries. As a result of this wide-ranging variety, it is imperative to note whether the assessment in question provides a standardised score or should only be considered in an isolated manner. One should take care to consider the structure and content of the assessment used in relation to the research question in order to determine its suitability regarding content and intended statistical analysis. This is particularly important when critiquing studies utilising non-standardised measures of mathematics over those taken from standardised batteries.

In summary, mathematical development begins before, and continues throughout, formal schooling. However, careful attention should be paid to the measures used to assess mathematics for research and educational purposes as their structure and content may influence the conclusions that can be drawn.

Theory of visuospatial working memory

Baddeley and Hitch (1974) first developed the concept of the visuospatial sketchpad as one of two slave systems in working memory, outlining its responsibility for storing and manipulating visual and spatial information. Researchers in the field of working memory have long since adopted the most recent revision of this model (Baddeley, 2000) as it has been demonstrated to accurately conceptualise findings (e.g. Andersson & Lyxell, 2007; Ashkenazi, Rosenberg-Lee, Metcalfe, Swigart, & Menon, 2013; Holmes & Adams, 2006) and to be robust to developments in understanding resulting from neuropsychological and dual task studies (e.g. Henson, 2002; Logie, 1995). As such, this model still holds as an appropriate explanation of working memory and is the model adopted by the studies included in this review. Currently, a focus on the emergence of simultaneous and sequential visuospatial working memory (see Mammarella et al., 2006 and Mammarella, Borella, Pastore, & Pazzaglia, 2013 for evidence of a double dissociation) is evident, in a move to understand the finer nuances of using visuospatial working memory as an academic predictor.

Simultaneous visuospatial working memory tasks are defined as such tasks whereby all information is presented to the participant at the same time (Mammarella et al., 2006). Following this presentation, the participant is asked to recall the positions of the stimuli they saw previously; an example of this type of task is the visual patterns task. In contrast, sequential tasks involve the presentation of stimuli in a sequence to the participant (as in Passolunghi & Mammarella, 2011). Participants are then required to recall the positions of the stimuli, typically in the correct order, as in the Corsi block task (Mammarella et al., 2006). There is evidence for the dissociation of these tasks (Mammarella et al., 2008), supporting the need for their independent investigation in order to assess their predictive power.

In line with these observations, a number of different visuospatial working memory tasks are used to tap into each of these components. As elements of standardised test batteries, a small number of visuospatial working memory tasks are standardised, however, a large proportion of the tasks used are designed for the purpose of the study in question. As such, it is imperative to assess the characteristics of the test in relation to the research question and statistical procedures applied before accepting the conclusions drawn from the results. This is of particular importance when studies employ a non-standardised visuospatial working memory measure.

Relationship between visuospatial working memory and mathematics

Importantly, visuospatial working memory is described by Ashkenazi et al. (2013) as a "source of domain general vulnerability in arithmetic cognition", indicating its position as one of a number of mechanisms in the brain which function to support learning in a broad range of areas. Such a definition also follows that knowledge is cumulative and so builds up over time to form our overall knowledge structure. As evidenced by the results of previous studies, age appears to be crucial to the extent of the involvement of visuospatial working memory in mathematics performance (Li & Geary, 2013), with the suggestion of a cyclical pattern of involvement between visuospatial working memory and verbal working memory. One could reasonably question the potential for an emerging relationship between novelty and mastery inherent in a cyclical relationship. Visuospatial working memory is more strongly predictive of mathematics performance in younger children (Holmes & Adams, 2006; Holmes, Adams, & Hamilton, 2008) which is, arguably, the period in which children are acquiring new mathematical skills at an increased rate. Therefore, it is possible that visuospatial working memory is employed to a greater extent during the procurement of new skills, and to a lesser extent once children achieve mastery of such skills (Andersson, 2008).

It may be possible to identify the age at which young children's mathematics ability is most strongly influenced by visuospatial working memory, and hence use this information to make predictions regarding future attainment. Research is currently moving to exploit this relationship further in order to train working memory to improve academic attainment (eg Holmes & Gathercole, 2014; see Sala & Gobet, 2017 for a review), however, this will only be possible when the intricacies of the relationship between the two factors are fully understood. Similarly, the potential to mediate vulnerability to mathematical difficulties as a result of poor working memory before they occur is hindered by a lack of detailed knowledge in this area. Before research in this area can progress, a clear representation of what is currently known in the literature is necessary. This review aims to provide this comprehensive picture.

In doing so, it is necessary to ensure that confounding factors are limited as far as possible. Often, studies employ tasks previously designed either to investigate a particular aspect of visuospatial working memory or mathematics, or those which form a component of a standardised battery. When appraising potential measures for a study, the age group for which the task was designed and, potentially standardised, is crucial. Only by considering the target age and that of the participants is it possible to make reasonable adjustments to prevent floor and ceiling effects. This is of particular importance when considering appropriate mathematics tasks as it is imperative that tasks administered align with concepts children have been exposed to through the curriculum. More leniency can be afforded to visuospatial working memory tasks as such tasks present fewer barriers to achievement should a child not have completed a similar task before. Further, given the nature of the research seeking to extend scientific understanding of the components of visuospatial working memory, novel tasks are required to access each component individually.

In summary, using visuospatial working memory as a means to predict pupil's future attainment in mathematics is a topic that has gained a significant amount of traction in recent years. Driven by the desire to improve academic performance, it is necessary to first ensure a clear understanding of the relationship between the two components before steps can be taken to use visuospatial working memory as a predictive tool.

Importance of this review

Given the relative infancy of this field of research, no other reviews concerning the relationship between visuospatial working memory and mathematics attainment have been identified. Szűcs (2016) completed a review on a similar field, identifying the relationships between subtypes of mathematical difficulties and elements of working and short-term memory. The available literature demonstrates both comparable and contrasting results which can only be adequately understood by appraising the results of the studies alongside their methodologies. In doing so it is possible to begin to explain the variations in results as features of the methodological differences. To this end, this review is necessary to consolidate the findings of previous research in order to provide a comprehensive understanding of the relationship between visuospatial working memory and mathematical performance. The results have a number of implications with regard to using visuospatial working memory as a predictive tool for future mathematical attainment, something which cannot be achieved without a streamlined understanding of the relationship central to forming these predictions, including, but not limited to, early intervention to improve attainment.

Objectives of the review

The aim of this review is to examine the literature surrounding the relationship between visuospatial working memory and mathematical attainment in children. Four key issues will be addressed; these are the influences of the age of the participants, the type of mathematics being assessed, the type of visuospatial working memory being assessed, and the nature of the tasks used (standardised/non-standardised). It is broadly understood that visuospatial working memory plays both an influential and predictive role in children's mathematical performance (Bull et al., 2008; Holmes & Adams, 2006), however, the exact relationship between these elements remains, as yet, unclear. The existing literature alludes to a number of factors that are influential in establishing a clear and coherent understanding of the role visuospatial working memory plays in mathematical development. This review will explore these potential confounds in a move to consolidate the existing knowledge on this issue. Focusing on the age of the participants, the components of mathematics being assessed, and the components of visuospatial working memory being measured, it is possible to begin to develop a more detailed understanding of the specific influences of each of these elements.

Method

Criteria for study inclusion

Studies eligible for inclusion in the analysis met all of the criteria outlined below.

Study design

Studies utilising all methodological designs were included in the review due to the nature of both the current literature and the review. Before inclusion, research must, however, have explicitly stated their intention to investigate the relationship between visuospatial working memory and mathematical attainment. Despite using studies with any design, before a study was included in the review, sufficient control and operationalisation of the variables must have been established. Testing should have been conducted in a controlled environment, with an emphasis on maintaining consistency between sessions in order to exert control in the absence of randomised control trials.

Type of participants

Studies of children attending mainstream schools, between the ages of 0 years and 16 years, were considered in this review. Three exclusion criteria applied: those investigating atypical populations, adults and young people over the age of 16 years old, and preterm children specifically. All ethnicities, socio-economic statuses, and genders were included.

Mathematics measures

The review included mathematics measures assessing elements of mathematics relating to numerical operations, mathematical problem solving, and/or mathematics as a whole; those utilising measures of number sense, numerosity, and other such related components were excluded. Whilst the majority of studies included in the review used standardised measures of mathematics, including the WIAT and WOND, a proportion used specifically designed measures. Studies of this nature were included so long as an observable, clear focus on one or more of the aforementioned components of mathematics was present. Where mathematics measures had been derived for the purpose of the study, this was typically in line with the curriculum outlined for children of the specified age in the given country.

Memory measures

Only studies published reporting visuospatial working memory as an explicit individual concept met the criteria for the review. Those reporting on working memory as a whole only, without further subdivision, were not included in the final sample. A number of standardised visuospatial working memory measures were employed, however, as a result of attempts to further subdivide visuospatial working memory, many measures were designed for the purposes of the study. As such, all measures specifically of visuospatial working memory were accepted.

Location of study

Studies may have been conducted in any country utilising an alphabetic language system to be eligible for inclusion, however, the final paper must be available in English. Only nations with alphabetic language systems were included due to the potential influence of logographic writing systems on the development of visuospatial working memory (Tan et al., 2001).

Additional criteria

Criteria were identified which led to the exclusion of a study. These exclusion criteria were studies concerning:

- Neuroimaging
- Mathematics anxiety
- Number sense/ numerosity
- Visual perception
- Working memory training
- Strategy use
- Interventions/ teaching methods
- Transcoding

Additional criteria for exclusion were texts from book chapters (serving only to summarise findings from included empirical studies) or other review articles.

Search methods for study identification (Search strategy)

Electronic searches

Searches were conducted of the databases listed below (using the 'all databases' option for each), with search terms defined as "visuospatial", "working memory", and "math*". Only articles where the full text was available were included. These terms were defined so as to identify all available studies that use these terms either in the title, abstract, or main body. Given the specificity of the desired work, simple, clearly defined search criteria were most appropriate.

- Web of Science
- JSTOR
- Science Direct
- Medline/ NCBI
- Scopus
- FirstSearch
- EBSCOhost

Search of other sources

Reference lists of the included papers were scrutinised to identify any further appropriate papers.

Data collection and analysis

Determining eligibility and data extraction

All data was extracted by the same author. Before any coding began, a stringent set of inclusion and exclusion criteria were clearly defined and periodic checks throughout data extraction were carried out to ensure criteria were adhered to at all times. Should a study be found to be ineligible upon full reading, the reasons for its exclusion were documented. Before beginning synthesis of results, the main statistic for each study was extracted and recorded.

Study coding categories

Any study that met the criteria for inclusion based on title, abstract, and full text reading was coded to extract the same information. This information included details regarding methodology, measures taken, participant details, the area of visuospatial working memory and mathematics being assessed, statistical method used and the main reported statistic, and a quality judgement of the study fit for the review (1 = very good fit; 2 = good fit; 3 = not very good fit e.g. Vanbinst, Ceulemans, Peters, Ghesquière and De Smedt (2018) = 1, very good fit, Caviola, Mammarella, Cornoldi and Lucangeli (2012) = 3, not very good fit). Once this information was compiled for each study, where not already given by the paper, an effect size was calculated and a quality judgement of the effect size calculation noted (1 = exact calculation, 2 = good approximation, 3 = rough approximation e.g. Campos et al (2013) = 1, exact calculation, Pina, Fuentes, Castillo and Diamantopoulou (2014) = 2, good approximation).

Determining effect sizes

Common effect sizes were calculated (r) for each paper so as to allow for direct comparison between studies. R was chosen as an appropriate effect size due to the assessment of overlap between the variables, rather than the difference between experimental groups. Where this was reported in the paper, this is the effect size reported, however, in other cases, this was calculated using an accessible effect size calculator from the Campbell Collaboration (Wilson, n.d.), alongside a second freely available calculator from

Psychometrica (Lenhard & Lenhard, 2016). These calculators allow calculation of effect sizes from a comprehensive range of study designs and so provide the most appropriate calculations of any given effect size. The use of two independent calculators allowed calculation of effect sizes from a greater variety of study design, where one calculator provided a means to convert a statistic in its absence from the other, as well as corroboration of calculations by using both calculators.

Dealing with missing data

In cases where sufficient data were not available to calculate effect sizes, where possible this information was calculated from other available data, for example the use of reported correlations from studies using multi-level models. As such, it was possible to calculate all of the required effect sizes, though the basis of such calculations on good approximations of the exact data was recorded in the quality judgements of the effect size calculations made for each study.

Assessment of heterogeneity

Due to the varied nature of the research available, it was unlikely that a meta-analysis would be possible. Studies included in the review demonstrated important differences between crucial aspects of their design, such as measures, methods used, and participants included. As a result, a thematic analysis using inferential statistics was concluded to be the most appropriate method for synthesis so as not to introduce error through drawing comparisons between dissimilar studies. An I^2 statistic of 89.81%, much higher than the recommended maximum of 25% when undertaking a meta-analysis, supports not completing a meta-analysis on the current data. Following findings by von Hippel (2015), it is important to consider the I^2 statistic in relation to the number of studies included in the meta-analysis, however, 35 studies should be sufficient to mitigate the potential for bias with a small number of studies.

Data synthesis

Due to the large variation in the studies included, and a number of confounding factors, including sample size and the use of unstandardized measures, a quantitative synthesis was completed using inferential statistics. As such, thematic analysis of the components of interest was completed, addressing issues of participant age, type of mathematics being assessed and component of visuospatial working memory being assessed, sample size, and the use of standardised measures.

Detecting and adjusting for publication bias

Most of the studies in this review concentrate on correlational relationships between the measured variables. As such, it is not unreasonable to suggest that publication bias may affect publication of these studies to a lesser extent as there is less of a drive to demonstrate a particular outcome. One must remain vigilant, however, as it remains the case that negative or more difficult to interpret results will be less easy to publish. In order to reduce publication bias introduced to this review, databases that include work such as theses and dissertations were also included when the literature search was conducted (see below for funnel plot). The non-significant Egger's regression ($p=0.21$), alongside the randomly distributed funnel plot, suggests there is no evidence of publication bias.

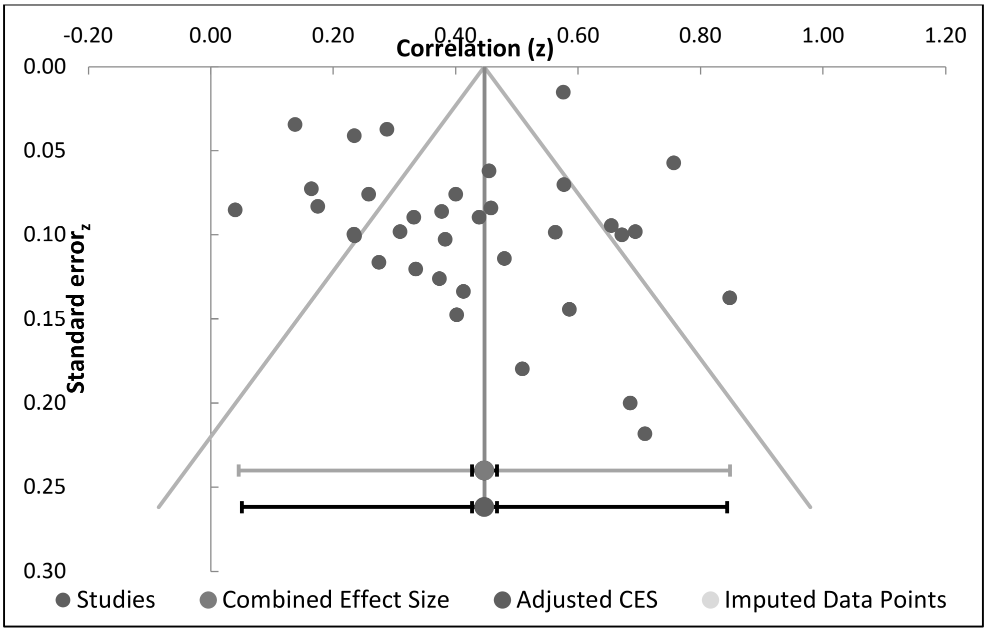


Figure 1. Funnel plot showing a random distribution, suggesting no evidence of publication bias.

Results

Description of the studies

Results of the search

The search of the above listed databases returned 590 records (search terms: "visuospatial" "working memory" and "math*"). Along with the electronic database searches, an additional 34 records were found as a result of the manual searches of reference lists completed.

52 of the records identified throughout the entire search process were duplicates and so were removed; 538 remained after this stage. Following screening of the titles and abstracts for irrelevant records, 469 records were excluded in accordance with the exclusion criteria, leaving 69 records. The remaining articles were read in full and the relevant data from 35 articles deemed appropriate, according to the inclusion and exclusion criteria, was extracted for analysis in the current review. Data was extracted from 35 articles in total.

Description of included studies

The studies included were conducted in a number of countries, and as such allow for a clearer understanding of the relationships between visuospatial working memory and mathematics performance globally, as opposed to solely in relation to the National Curriculum followed in the UK. Further, the broad age range of participants allows for an understanding to be established regarding the potential fluctuations in this relationship as children mature and undergo more formal schooling in mathematics.

The studies included adopted a number of methodological designs, however, no specific inclusion and exclusion criteria were defined regarding methodology as it was anticipated that a broad range of designs would be used. As a result, all study designs were included. Owing to the variety of methodological designs used, the resulting statistical analyses employed by the included studies also varied greatly. Whilst a vast majority of studies employed, at least as part of their analysis, ANOVA, correlation, and regression techniques, additional techniques including factor analysis, structural equation modelling, and multi-level modelling were used to further explain the data gathered. For this review, the main result from each study was converted to a correlation co-efficient, r, in order to make accurate comparisons between studies.

Quantitative synthesis of results

Overall findings

Sufficient data were provided by each of the studies included in the analysis to be included in the quantitative synthesis. As noted above, a full meta-analysis of the data was not conducted due to the vast differences inherent in the study designs. It was deemed that there were insufficient similarities within the studies for a meta-analytical comparison to be tangible due to the impact on the subsequent interpretation of the results. Rather, inferential statistics were employed, where possible, to achieve an objective assessment of the relationships within the data. Analyses were conducted on a number of subsections of the data by way of identifying the possible sources of the aforementioned heterogeneity in order to better understand the relationship between visuospatial working memory and mathematics performance.

As previously mentioned, effect sizes were calculated based on the most relevant result to the review topic, with an average effect size taken in situations when more than one statistic was equally relevant. Since all effect sizes calculated resulted from different studies, they can be considered independent. The results demonstrated an overall positive relationship between visuospatial working memory and mathematics, as evidenced by the forest plot in figure 2, below. From the funnel plot, figure 1, publication bias appears minimal in the studies available on this subject.

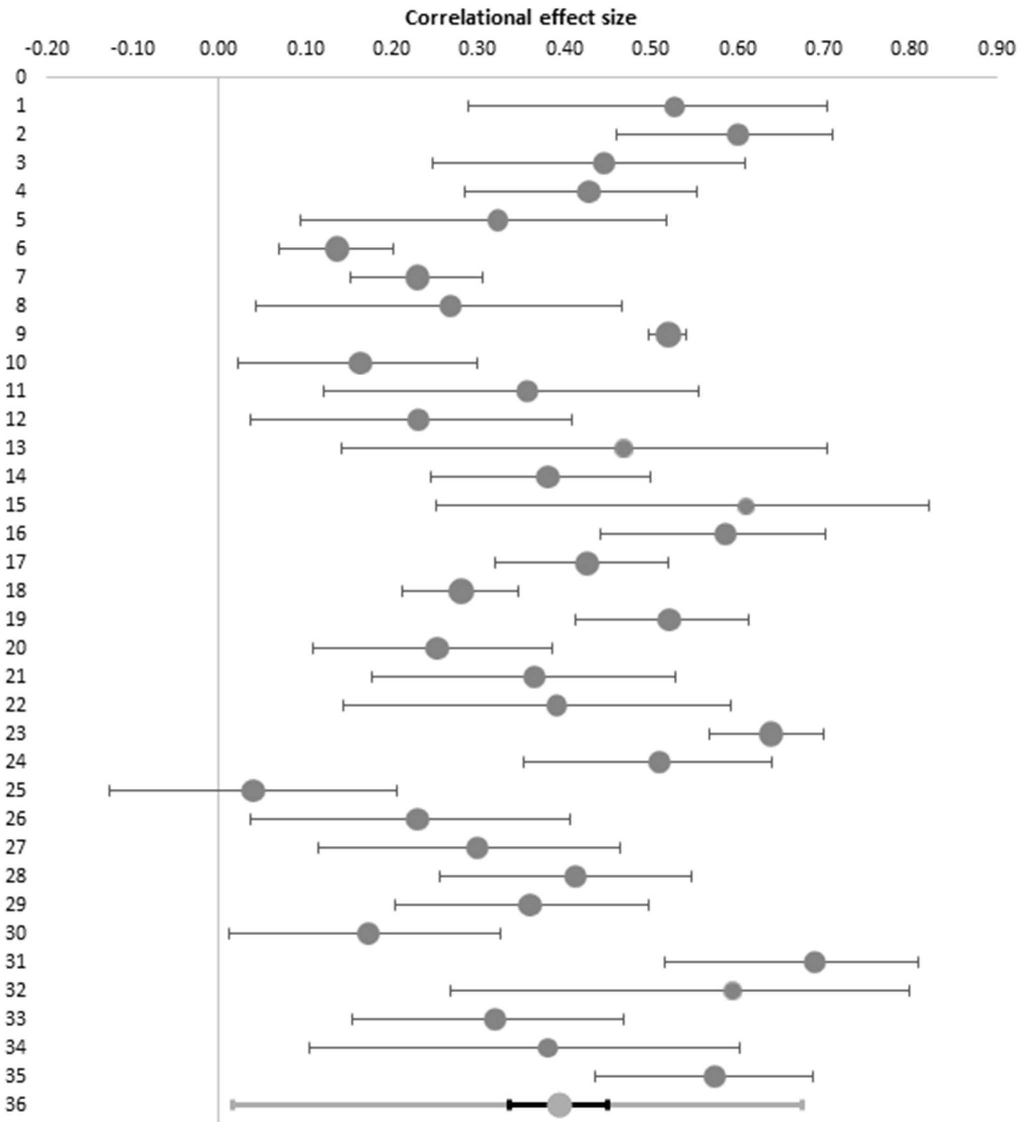


Figure 2. Forest plot showing an overall significant positive relationship between visuospatial working memory and mathematics.

Subsection analysis

Sample size

Upon investigation, a relationship between sample size and effect size is present within the data. This section strives to investigate this relationship further to understand the potential ways in which sample size may influence the effect sizes found.

Larger effect sizes appear concurrent with smaller sample sizes ($r_s = .340$, $p = .046$), as previously demonstrated in the literature as a common phenomenon and indicative of potential publication bias (Kühberger, Fritz, & Scherndl, 2014; Levine, Asada, & Carpenter, 2009). The correlation between sample size and effect size is stronger following removal of one study with an extremely large sample size (Van de Weijer-Bergsma, Kroesbergen, & Van Luit, 2015; r_s = .404, p = .018), however caution should be applied when interpreting this finding due to issues of statistical power (Button et al., 2013). A medium-large effect size resulting from the study with the largest sample size (4337; Van de Weijer-Bergsma et al., 2015) sits comfortably within the range of effect sizes, hence reducing the potential influence of sample size on effect size (Button et al., 2013).

Sample sizes were divided into two groups for further analysis: small (mean = 115.82, sd = 65.84, lower bound = 24, upper bound = 308) and large (mean = 1627.75, sd = 1809.21, lower bound = 597, upper bound = 4337). No significant difference was found between the two groups (t(33) = -1.357, p-.184), suggesting a lesser influence on sample size than indicated by Button et al. (2013). Finally, once negative effect sizes were transformed into positive (via a reflection of the original due to the +/- difference resulting from the labels assigned to M1/M2), they did not deviate from the core cluster and, hence, show no significant differences from the remaining effect sizes.

Type of mathematics

Approximately equal numbers of studies investigated Numerical Operations and both Numerical Operations and Mathematical Reasoning (17 and 16, respectively), however only two studies considered purely Mathematical Reasoning. Interestingly, the largest mean effect size was produced by studies concerning Mathematical Reasoning (mean=0.49), with studies using small-average samples (n=30 and n=103), suggesting that this result cannot be explained by sample size alone. Those studies investigating both types of mathematics demonstrated the next largest effect size (mean=0.43), followed by Numerical Operations only (mean=0.35), indicating that visuospatial working memory may be more of an influencing factor in Mathematical Reasoning than Numerical Operations. Despite the aforementioned differences being present in the data, the between group differences were not statistically significant (F(2)=1.380, p=0.266). It is evident from the data that Numerical Operations and a combination of both Numerical Operations and Mathematical Reasoning showed greater spread of effect sizes (range=0.55 and 0.52, respectively), though only 2 studies looked at Mathematical Reasoning alone (range=0.20). It is to be expected that the range of effect sizes resulting from studies of Mathematical Reasoning would have been greater if more studies had investigated Mathematical Reasoning alone.

Two studies (Maennamaa, Kikasb, Peets, & Palu, 2012; Wiklund-Hörnqvist et al., 2016) investigated both types of mathematics using large samples, which may have skewed the average effect size generated for this subgroup as 13 studies used small samples. However, as suggested by Button et al. (2013), it may be the case that these larger samples provide the power to detect effects within the data and increase the likelihood that statistically significant results are reflective of true effects. Both studies assessing Mathematical Reasoning (Campos et al., 2013; Passolunghi & Mammarella, 2010) had only small sample sizes (103 and 59, respectively) and as such, according to Button et al. (2013), the large effect sizes may be less likely to be representative of the true population effect.

Type of visuospatial working memory

Studies were broken down according to the type of visuospatial working memory they assessed: simultaneous, sequential, or both. The largest mean effect size was observed for studies concerning both simultaneous and sequential working memory (mean=0.44), followed by sequential (mean=0.37), and simultaneous (mean=0.25). Whilst this difference is marginally non-significant (F(2)=2.727, p=0.081), it is suggestive of a bias in the level of influence of each type of visuospatial working memory on mathematics performance.

The largest range of effect sizes can be seen in the data for both types of visuospatial working memory (range=0.65), with smaller ranges seen for sequential and simultaneous (range=0.44 and 0.29, respectively). Such a finding alludes to other influencing factors in studies measuring both types of visuospatial working memory due to the large range of effect sizes displayed. Further, it may suggest the more stable development of simultaneous visuospatial working memory by the age of children included in these studies (5 and 6 years, respectively, for both types of visuospatial working memory and simultaneous only). All four studies involving large sample sizes (Maennamaa et al., 2012; Mix et al., 2016; Van de Weijer-Bergsma et al., 2015; Wiklund-Hörnqvist et al., 2016) concerned both types of visuospatial working memory, which may explain, in part, the large range of effect sizes for this category (14 used small samples), whereas all studies concerning only simultaneous or sequential visuospatial working memory used small sample sizes.

Type of visuospatial working memory was measured alongside type of mathematics to ascertain further detail on more specific relationships between the two components. No studies investigated the influence of sequential visuospatial working memory on Mathematical Reasoning, highlighting a gap in the research requiring additional investigation. An ANOVA showed no significant effects when using type of working memory and type of mathematics as fixed effects. Simultaneous visuospatial working memory shows the lowest mean effect sizes for both Numerical Operations and both types of mathematics (mean=0.28 and 0.23, respectively), suggesting that simultaneous visuospatial working memory has the smallest influence on mathematical performance in these areas of mathematics (simultaneous visuospatial working memory alone was not measured for Mathematical Reasoning). The largest mean effect size (mean=0.49) was identified for both types of visuospatial working memory in Mathematical Reasoning tasks. A large mean effect size here implies a large influence of visuospatial working memory in Mathematical Reasoning tasks, in line with the additional demands of such tasks, however, only two studies (Campos et al., 2013; Passolunghi & Mammarella, 2010) measured this combination and so caution should be exercised when generalising the result. As may be expected, studies measuring both types of visuospatial working memory showed the largest mean effect size, regardless of the type of mathematics being investigated (Numerical Operations, Mathematical Reasoning, or both). One potential explanation for this may be the need to combine information and/or the complexity of the tasks used, particularly in the case of studies assessing Mathematical Reasoning and both types of mathematics.

Age of participants

The age of participants at the beginning and end of each study was extracted for in depth analysis. Neither showed a significant correlation with effect size ($r_s(35)=-0.025$, $p=0.885$; $r_s(35)=-0.178$, $p=0.307$, respectively). The mean age at the beginning of the included studies was 7.89 years, with a range from 4-15 years (sd=2.44). Once all studies had reached their conclusion, the mean age showed an increase to 9.86 years, ranging from 7-16 years (sd=2.35). Studies concerning Numerical Operations showed the lowest mean age at the beginning of the study (mean=7.29 years; range=4-12), therefore, it is conceivable that the effect sizes generated for this type of mathematics might be affected by the involvement of such young participants. Further, the involvement of younger participants in studies surrounding Numerical Operations aligns with methods for teaching mathematics, whereby arithmetic skills are taught before any reference to word problems or other such questions linking to Mathematical Reasoning. Studies investigating both types of mathematics involved the largest age range of participants (mean=8.44 years, range=5-15 years). Such a large range of ages, and the combination of styles of mathematics questions, may have influenced the effect sizes collected due to the demand of the questions, particularly those relating to Mathematical Reasoning. Mathematical Reasoning questions requiring a high level of proficiency in reading may have proven particularly detrimental to young children's Mathematical Reasoning scores. A further potential influence on results concerns whether an age appropriate/ standardised measure of mathematical ability was taken to assess performance.

Studies assessing sequential working memory involved the youngest mean age of participant (mean=6.85 years), with an age range of 4-9 years. The mean age at the beginning of studies for those assessing simultaneous working memory was non-significantly higher than sequential working memory (mean=7.25 years, p=0.955). It would not be expected that an age difference as small as can be observed in the given data would have a significant impact on working memory and mathematics performance. The largest observable age range can be seen for studies examining both types of working memory (range=5-15 years), with a mean age of 8.78 years. This is also the group of studies with the oldest mean age. All studies using older children, of secondary school age, fall into this category, which would be expected to influence the results as it is expected that older children will have a larger working memory capacity (Gathercole, Pickering, Ambridge, et al., 2004).

Standardised measures

Studies were examined according to whether they had employed a standardised measure of visuospatial working memory or not. The mean effect size for studies using a standardised measure was higher, but not statistically significantly so, than that found for those using non-standardised measures (mean=0.40 and mean=0.38, respectively; $t(33)=0.212$, $p=0.833$). A non-significant finding here indicates that standardised and non-standardised measures appear to be equally effective at measuring visuospatial working memory in relation to mathematics performance. Further, there was no significant relationship between the size of the sample used and the use of a standardised measure ($t(12.154)=-1.143$, $p=0.275$). As such, the use of a standardised measure and sample size are unlikely to have a compound influence on effect size.

Studies were then examined according to their use of a standardised mathematics measure. The mean effect size gathered for studies using a standardised mathematics measure (mean=0.44) was significantly higher than those using unstandardized measures (mean=0.25, $t(33)=3.587$, $p=0.001$). Such a finding highlights the importance of using standardised mathematics measures in order to uncover the true extent of any relationship between mathematics performance and visuospatial working memory. As with measures of visuospatial working memory, the data do not show a significant relationship between the size of the sample used and the use of a standardised measure ($t(33)=0.125$, $p=0.901$). Therefore, the size of the sample is unlikely to have a compound effect on the already significant influence of the use of a standardised measure.

Overall, the results indicate the importance of using a standardised measure of mathematics when investigating the relationship between visuospatial working memory and mathematics performance. However, they also suggest that the use of a non-standardised measure of visuospatial working memory does not necessarily prove detrimental to the integrity of the study.

Table 1. *Number of study participants, age of participants, mathematics measures used, and visuospatial working memory measures used for each study included in the analysis*.

Author(s)	Date	N	Age (in years)	Mathematics measures	Visuospatial working memory measures
Vanbinst et al.	2018	51	5-8	Standardised addition and subtraction task	Corsi block task
Vandenbroucke et al.	2018	107	6-7	Standardised achievement test	Dot matrix; block recall; odd one out; Mr X
Bresgi, Alexander, & Seabi	2017	80	7-8	Group mathematics test	Spatial recall; spatial processing recall
Li & Geary	2017	145	12-15	Weschler individual achievement test (WIAT)	Block recall; mazes memory
Mammarella, Caviola, Giofrè, & Szűcs	2017	72	9-10	AC-MT 11-14 standardised arithmetic battery, AC-FL, BDE-2 battery	Visual memory houses/balloons, spatial-simultaneous

					and spatial-sequential matrices
Mix et al.	2016	854	6-11	Place value, word problems, calculation, missing term problems/ algebra, number line estimation, fractions	Adaptation of dot matrix
Wiklund-Hörnqvist et al.	2016	597	9	Swedish national test in mathematics for grade 3 pupils	Adaptation of WISC-IV block span
Soltanlou, Pixner, & Nuerk	2015	77	8-11	Multiplications	Corsi block task forwards/ backwards
Van de Weijer-Bergsma et al.	2015	4337	5-10	Arithmetic tempo test	Lion game
Martin, Cirino, Sharp, & Barnes	2014	193	6-7	Procedural counting; conceptual counting; symbolic number identification (K); small sums addition & subtraction; WJ-3	Adaptation of dot matrix

				calculation; WRAT-3 arithmetic; WJ-3 applied problems subtest; single digit story problems (1st grade)	
Nath & Szücs	2014	66	7	Numerical operations subtest (WIAT)	Dot matrix; odd one out
Pina et al.	2014	102	9-10	Fluency and quantitative concepts tests from Spanish version of WJ-III ACH; arithmetic test from Spanish WISC	Computerised Corsi block task forwards and backwards
Ashkenazi et al.	2013	34	7-9	Numerical operations and mathematical reasoning subtests (WIAT)	Block recall
Li & Geary	2013	177	6-11	WIAT	Block recall; mazes memory

Szucs, Devine, Soltesz, Nobes, & Gabriel	2013	24	9	Mathematical assessment for learning test; numerical operations subtest (WIAT)	Dot matrix; odd one out
Campos et al.	2013	103	8-9	Arithmetic word problems; measurements	Block recall; mazes memory
Caviola et al.	2012	263	8-9	Standardised arithmetic battery	Dot matrix
Maennamaa et al.	2012	723	8-9	Maths test designed in line with the third grade Estonian curriculum	Figure recognition test
Alloway & Passolunghi	2011	206	7-8	AC-MT, WOND	Dot matrix; mazes memory; block recall; odd one out; Mr X; spatial recall
Geary	2011	177	7-10	Numerical operations subtest (WIAT)	Block recall; mazes memory
Meyer, Salimpoor,	2010	98	7-8	Numerical operations and mathematical	Block recall

Wu, Geary, & Menon				reasoning subtests (WIAT)	
Passolunghi & Mammarella	2010	59	9	12 item standardised mathematics test; WRAT calculation subscale	Corsi block task; spatial matrix; houses recognition task; pathway span
Alloway & Alloway	2009	308	5-9	Weschler objective numerical dimensions (WOND)	Odd one out; Mr X; spatial recall; dot matrix; mazes memory; block recall
De Smedt et al.	2009	106	6-7	Flemish Student Monitoring System	Block recall; visual pattern task
Andersson	2008	141	9-11	Horizontally presented addition, subtraction, and multiplication problems; arithmetic fact retrieval	Visual matrix; Corsi block task
Bull et al.	2008	104	4-7	Performance indicators in primary school (PIPS)	Corsi block task forwards and backwards

Holmes et al.	2008	107	7-10	Maths tests designed to test the 4 elements of the national curriculum	Visual patterns test; block recall
Kyttälä & Lehto	2008	128	15-16	The Mathematics Test (Finland)	Visual patterns test; Corsi block task; mental rotation
Andersson & Lyxell	2007	138	9-10	Simple addition	Dot matrix; Corsi block task
Holmes & Adams	2006	148	7-10	Maths tests designed to test the 4 elements of the national curriculum	Mazes memory
Bayliss, Jarrold, Baddeley, & Gunn	2005	56	7-9	NFER-Nelson mathematics	Target search; adaptation of dot matrix
D'Amico & Guarnera	2005	28	9-11	ABCA	Matrix task; corsi block task
Jarvis & Gathercole	2003	128	10-14	National curriculum composite results	Visual patterns test; dot matrix; spatial

					span task; odd one out task
Maybery & Do	2003	49	9-10	Wood & Lowther Easymark Diagnostic Mathematics Test	Fixed spatial span; running spatial span
Reuhkala	2001	115	15-16	National mathematics test	Matrix pattern task; Corsi block task

Table 2. *Effect size (r), confidence interval for effect size, type of mathematics, and type of visuospatial working memory for each study included in the analysis*.

Author(s)	Date	r	Lower CI	Upper CI	Type of mathematics	Type of visuospatial working memory
Vanbinst et al.	2018	0.527	0.294	0.701	Numerical operations	Sequential
Vandenbroucke et al.	2018	0.600	0.463	0.709	Numerical operations & mathematical reasoning	Sequential
Bresgi et al.	2017	0.446	0.251	0.606	Numerical operations	Sequential
Li & Geary	2017	0.428	0.285	0.553	Numerical operations	Simultaneous & sequential
Mammarella et al.	2017	0.323	0.099	0.516	Numerical operations	Simultaneous & sequential
Mix et al.	2016	0.137	0.071	0.202	Numerical operations & mathematical reasoning	Simultaneous

Wiklund-Hörnqvist et al.	2016	0.230	0.153	0.305	Numerical operations & mathematical reasoning	Sequential
Soltanlou et al.	2015	0.268	0.047	0.464	Numerical operations	Sequential
Van de Weijer-Bergsma et al.	2015	0.520	0.498	0.541	Numerical operations	Sequential
Martin et al	2014	0.163	0.022	0.297	Numerical operations	Sequential
Nath & Szücs	2014	0.357	0.126	0.551	Numerical operations	Sequential
Pina et al.	2014	0.231	0.038	0.407	Numerical operations	Sequential
Ashkenazi et al.	2013	0.469	0.156	0.697	Numerical operations & mathematical reasoning	Sequential
Li & Geary	2013	0.380	0.246	0.500	Numerical operations	Simultaneous & sequential

Szucs et al.	2013	0.610	0.274	0.813	Numerical operations & mathematical reasoning	Simultaneous & sequential
Campos et al.	2013	0.586	0.443	0.700	Mathematical reasoning	Sequential & simultaneous
Caviola et al.	2012	0.425	0.321	0.520	Numerical operations	Simultaneous
Maennamaa et al.	2012	0.280	0.211	0.346	Numerical operations & mathematical reasoning	Simultaneous
Alloway & Passolunghi	2011	0.520	0.413	0.614	Numerical operations	Simultaneous & sequential
Geary	2011	0.253	0.109	0.386	Numerical operations	Simultaneous & sequential
Meyer et al.	2010	0.365	0.180	0.526	Numerical operations & mathematical reasoning	Sequential

Passolunghi & Mammarella	2010	0.391	0.150	0.588	Mathematical reasoning	Simultaneous & sequential
Alloway, Gathercole, Kirkwood, & Elliott	2009	0.639	0.567	0.701	Numerical operations & mathematical reasoning	Sequential & simultaneous
De Smedt et al.	2009	0.510	0.354	0.639	Numerical operations & mathematical reasoning	Simultaneous & sequential
Andersson	2008	0.040	-0.126	0.204	Numerical operations	Simultaneous & sequential
Bull et al.	2008	0.230	0.039	0.405	Numerical operations	Sequential
Holmes et al.	2008	0.300	0.117	0.463	Numerical operations & mathematical reasoning	Simultaneous & sequential
Kyttälä & Lehto	2008	0.412	0.257	0.547	Numerical operations &	Simultaneous & sequential

					mathematical reasoning	
Andersson & Lyxell	2007	0.360	0.205	0.497	Numerical operations	Simultaneous & sequential
Holmes & Adams	2006	0.173	0.012	0.325	Numerical operations & mathematical reasoning	Simultaneous
Bayliss et al.	2005	0.690	0.521	0.806	Numerical operations & mathematical reasoning	Simultaneous & sequential
D'Amico & Guarnera	2005	0.595	0.285	0.792	Numerical operations	Simultaneous & sequential
Jarvis & Gathercole	2003	0.320	0.155	0.468	Numerical operations & mathematical reasoning	Simultaneous & sequential
Maybery & Do	2003	0.381	0.112	0.598	Numerical operations &	Sequential

					mathematical reasoning	
Reuhkala	2001	0.574	0.437	0.685	Numerical operations & mathematical reasoning	Simultaneous & sequential

Discussion

Systematic review results summary

This review concerned 35 independent studies, following thorough examination of each document to ensure no overlaps between studies were present. The review was conducted with the aim of producing a comprehensive overview of the current knowledge base relating to the relationship between visuospatial working memory and mathematics. The included studies comprised of a number of designs and involved a variety of assessments of both mathematics and visuospatial working memory. Whilst this is a relatively small sample of studies for the purposes of a review, there were a sufficient number to conduct further analysis. A forest plot and funnel plot (figures 1 and 2) were generated to give an overview of the data before inferential statistics were applied in the absence of a meta-analysis.

The number of studies analysed for this review is reflective of the current understanding of the relationship between visuospatial working memory and mathematics. Research remains in its relative infancy, therefore, the intricacies of the relationship are as yet unknown. For example, the earliest study in this review demonstrates the first published study documenting the specific relationship as taking place in 2001 (Reuhkala, 2001).

No other systematic reviews on this area of the research have been published, to our knowledge, up to the date of writing, hence there is great scope for collating the findings of the research to date. As such, it is not possible to draw comparisons with the findings of reviews of other aspects of this area of research. The lack of reviews previously completed in this area indicates the need to develop a comprehensive understanding of the given relationship before continuing with further research.

Quantitative analysis results summary

The review results highlight the importance of a sufficiently large sample in order to detect any effect within the data and accurately determine its significance, as evidenced by the negative correlation identified between effect size and sample size. The inclusion of only two studies exploring solely Mathematical Reasoning demonstrates an evident lack in the literature of such focused work, however, a further 16 studies investigated Mathematical Reasoning in conjunction with numerical operations.

From the evidence, it appears that numerical operations and mathematical reasoning are both influenced to a similar extent by visuospatial working memory, however, the level of influence within each of these types of maths is variable. The greatest variation can be seen for numerical operations. A bias in the amount of influence of the type of visuospatial working memory is suggested from the data. Nevertheless, once the type of mathematics being assessed is included in the analysis, the difference is not significant. Further, age did not have a significant impact on the effect sizes generated, nor did the use of a standardised visuospatial working memory measure. On the contrary, the use of a standardised mathematics measure resulted in a significantly larger effect size. One possible reason for such a difference may be the design of standardised measures to rigorously assess specific areas of mathematics and address all areas of the curriculum.

Quality of the evidence

590 studies were screened before arriving at the final sample of 35, suggesting that a sufficiently scoping search was completed to identify all relevant available literature, in line with the inclusion criteria. This suggestion is supported as all relevant studies were available in full.

As previously mentioned, the studies included employ a number of designs, measures, and methods of analysis. This emphasises the need to apply caution when attempting to directly compare across studies. However, sufficient data was provided within each manuscript to allow for the calculation of effect sizes, thus allowing less problematic, direct comparisons due to the common scale. Comparisons have also been drawn regarding the variance accounted for, in order to examine the extent of the influence, as well its significance, so as to reduce the probability of making type 2 errors, given the potential for the small studies included to be underpowered. As a result, the conclusions drawn from the data in this review seem relatively robust.

Conclusions

This review analysed the available literature on the relationship between visuospatial working memory and mathematics and proposes that the type of visuospatial working memory and mathematics being assessed do not have significant influence, however, the use of a standardised mathematics measure demonstrates significant influence on the effect size generated. Overall a positive influence of visuospatial working memory on mathematics attainment is evident.

Implications for research

The findings presented above suggest the greatest implications for those seeking to develop visuospatial working memory research in relation to mathematics performance. Since there is the suggestion that the use of a standardised mathematics measure significantly influences the estimation of the level of effect, researchers ought to be cautious of devising their own measures of mathematics attainment where a suitable standardised measure is available.

There is a great deal of scope for further research suggested by the findings of this review, as well as the gaps in the research identified throughout. Additional research is necessary to determine the stability of the relationship as identified over the years children spend at school. For example, in order for preventative measures for mathematical difficulties to be devised, it is first necessary to understand the intricacies of the relationship. Additionally, further research should seek to identify whether the relationship identified throughout is specific to components of mathematics, or whether the explanation satisfies mathematics in general.

7

Study 1 Introduction

The systematic review presented above highlighted a number of areas that required further investigation upon continuation of this project. Spatial-simultaneous and spatial-sequential working memory appear to show different relationships with mathematics, with spatial-sequential demonstrating a stronger relationship to mathematics than spatial-simultaneous (difference in r=.12). It is important to consider that the review covered a broader range of school-aged children, and as such it is necessary to further investigate this relationship with regard to the target age range for this project. In order to establish a more thorough understanding of how the subcomponents of visuospatial working memory relate to mathematics, specifically in primary school aged children, we decided to conduct a study that distinguished between the two. In doing so, it was possible to look into the differential influence on maths of spatial-simultaneous and spatial-sequential working memory. We further subdivided spatial-sequential working memory to differentiate between those tasks that required participants to adhere to the given order of stimuli during the recall phase, and those that did not. This ensured a fully crossed design. Year three (7-8 year olds) was chosen due to the ages at which standardised testing is conducted within schools, since Year three is generally a year group with less pressure attached as no external testing is required in this year group. Further, using Year three as the test group left scope for extending the findings into younger children in order to understand the malleability of the relationship developmentally. The paper that follows is published in the Quarterly Journal of Experimental Psychology (Allen, K., Giofrè, D., Higgins, S., & Adams, J. (2020). Working memory predictors of written mathematics in 7-to 8-year-old children. *Quarterly Journal of Experimental Psychology*, *73*(2), 239–248.; Appendix J). It was authored in collaboration with Dr. David Giofrè, and allowed me to develop more advanced analysis skills, including using R for data analysis and using structural equation modelling as a technique to understand the data beyond the capabilities of standard regression models. We established the collaboration following meeting at the Annual Working Memory Discussion Meeting in June 2018 and have maintained the collaboration since. In this instance, I had designed and conducted the study, Dr. David Giofrè joined the project during the analysis phase.

8

Study 1: Working Memory Predictors of Written Mathematics in 7-8 Year Old Children

Abstract

There is extensive evidence for the involvement of working memory in mathematical attainment This study aims to identify the relative contributions of verbal, spatial-simultaneous, and spatial-sequential working memory measures in written mathematics. Year 3 children (7-8 years of age, n=214) in the UK were administered a battery of working memory tasks alongside a standardised test of mathematics. Confirmatory factor analyses and variance partitioning were then performed on the data to identify the unique variance accounted for by verbal, spatial-simultaneous, and spatial-sequential measures. Results revealed the largest individual contribution was that of verbal working memory, followed by spatial-simultaneous factors. This suggests the components of working memory underpinning mathematical performance at this age are those concerning verbal-numeric and spatial-simultaneous working memory. Implications for educators and further research are discussed.

Introduction

There is some discrepancy in the literature with regard to the proportional influence of components of the Baddeley and Hitch working memory model (1974) on mathematics achievement. Whilst there are suggestions of a stronger influence of visuospatial working memory (e.g., Caviola, Mammarella, Lucangeli, & Cornoldi, 2014; Clearman, Klinger, & Szucs, 2017; Holmes, Adams, & Hamilton, 2008; Li & Geary, 2017), there is also evidence of developmental shifts in the respective contributions and the potential for a cyclical relationship (e.g., Li & Geary, 2013; Soltanlou, Pixner, & Nuerk, 2015; Van de Weijer-Bergsma, Kroesbergen, & Van Luit, 2015). Additionally, there is some evidence for a greater influence of verbal working memory (e.g., Wilson & Swanson, 2001) on mathematics. Visuospatial working memory is implicated in mathematics performance in a number of areas, including, but not limited to arithmetic (Ashkenazi et al., 2013; Caviola et al., 2012; Passolunghi & Cornoldi, 2008), word problem solving (Swanson & Beebe-Frankenberger, 2004; Swanson & Sachse-Lee, 2001; Zheng, Swanson, & Marcoulides, 2011), and geometry (Giofrè, Mammarella, & Cornoldi, 2014a; Giofrè, Mammarella, Ronconi, & Cornoldi, 2013), as well as mathematical difficulties (Andersson & Lyxell, 2007; D'Amico & Guarnera, 2005; McLean & Hitch, 1999; Passolunghi & Cornoldi, 2008; Szucs et al., 2013). It is, therefore, important to understand the intricacies of this relationship in order to mediate difficulties associated with mathematics to the fullest extent possible.

Some authors argued that the visuospatial working memory system is not unitary (e.g., Logie, 1995). An alternative approach that has recently received some support is one that distinguishes between spatial-sequential tasks requiring the recall of a sequence of spatial locations, and spatial-simultaneous tasks demanding the recall of an array of simultaneously-presented locations (see Cornoldi & Vecchi, 2003; Mammarella, Borella, Pastore, & Pazzaglia, 2013; Mammarella, Caviola, Giofrè, & Szűcs, 2018).

Mammarella et al. (2006, 2018) identified a double dissociation between spatial-simultaneous and spatial-sequential working memory, which has been further investigated for its relationship with mathematics, thus providing reason for differentiating between spatial-simultaneous and spatial-sequential formats of visuospatial working memory tasks. Since spatial-simultaneous and spatial-sequential visuospatial working memory can be uniquely affected in visuospatial learning difficulties, it is logical that these two components may demonstrate differential relations with mathematics attainment in young children.

Various measures are available for assessing mathematical performance, ranging from single-step calculations to multi-step contextual story problems. A number of these measures have been standardised for their use with children within a particular age range (e.g., Wechsler Individual Achievement Test, Pearson Clinical, 2017), however, a large number of the measures used are measures derived by researchers for the purpose of research. Measures designed for research purposes should be considered carefully when applying the findings to any context other than that it was originally designed for since direct comparisons are not possible from unstandardised data. Furthermore, such measures can lead to concerns regarding reliability and validity since the number of applications of the measures is generally fewer than that of standardised measures. To combat these issues, a standardised written mathematics measure was used in this study to ascertain how children performed compared to age norms. The measure is designed to map on to current England and Wales SATs papers and so is directly related school attainment data.

The principal aim of this study is to examine the relationship between different working memory components and mathematics attainment. Here we aimed to further this knowledge by identifying the unique contributions of verbal, spatial-simultaneous, and spatial-sequential factors to written mathematics in Year 3 children (7-8 years of age). In doing so, this knowledge will allow us to understand more deeply the predictive nature of this relationship and understand where best to target preventative measures for mathematics difficulties, for example by identifying the age group most likely to benefit from an intervention. This age group was chosen based on previous evidence highlighting a stronger influence of visuospatial working memory on mathematics attainment in this age group (Holmes & Adams, 2006; Holmes et al., 2008). The age group chosen also aligns with a period of intensive skill acquisition; a time when visuospatial working memory is most likely employed (Andersson, 2008).

Spatial-sequential tasks requiring order during the recall phase, as well as those that do not require order during recall, were used in order to ensure the model was fully crossed. The main research question being asked was “how do the subcomponents of working memory relate to the performance of written mathematics?”. Previous meta-analytic findings indicate different subcomponents of working memory do not tend to make different contributions to mathematical performance (Peng, Namkung, Barnes, & Sun, 2016). Such a finding, however, might be determined by a heterogeneous number of measures in use in different studies and by the fact that the aforementioned meta-analysis did not distinguish between simultaneous and sequential subcomponents of working memory. In addressing this issue, a recent systematic review by Allen, Higgins and Adams (2019) identified no influence of spatial-sequential versus spatial-simultaneous working memory on mathematical performance. Similarly to Peng, Namkung, Barnes and Sun (2016), this review compared studies with a wide range of measures both for mathematics and working memory. Further, verbal components of working memory were not considered, which may have influenced the results. This work will expand on the understanding of previous papers by including the unique contributions of spatial-simultaneous and spatial-sequential measures to children’s mathematics.

Method

Participants

The sample initially included 214 7-8 year old children. Some children were absent during the second administration and were excluded from the final sample. The final sample included a total of 197 children (95 male and 102 female, M age = 95.99 months, SD = 3.63). An opportunity sample of Year 3 pupils in each of the five schools was used, using opt-out parental consent to reduce bias in the sample (Krousel-Wood et al., 2006). The study was approved by the School of Education Ethics Committee at the University of Durham. Parental consent was obtained. Children with special educational needs, intellectual disabilities, or neurological and genetic conditions were not included in the study.

Design & Procedure

All children were tested individually in a quiet area of their school. The six working memory measures were administered in a randomised order so as to reduce the influence of rehearsal or fatigue (α = .80). However, the size of the grids used in the derived measures of visuospatial working memory were administered in a fixed order (3 × 3 then 4 × 3, and 4 × 3 then 4 × 4, for spatial-sequential and spatial-simultaneous, respectively). A correlational design was adopted to explore the relationships between visuospatial working memory and maths performance. Measures were administered as per the administration instructions provided with the WMTB-C where standardised measures were used. Where measures were derived for the purposes of the study, administration procedures paralleled those set out for standardised measures. The mathematics test was presented in paper format. Children could ask for a question to be read aloud in order to not place children of lower reading ability at a disadvantage.

Measures

Verbal working memory

Working Memory Test Battery for Children (WMBT-C)*:* Three subtests of the WMTB-C were administered to children: digit recall (children recall a list of digits presented to them verbally), backwards digit recall (children recall a list of digits presented to them verbally in reverse order), and counting recall (children count aloud the number of dots on a page then recall the list of totals, in the correct order, once all pages in the sequence have been counted). All subtests were administered in accordance with the instructions set out for the WMTB-C, hence sequences were presented at a rate of one item per second. Blocks of six trials of each sequence length were employed, however, following four correct trials, testing moved on to the next block. Testing was discontinued following three mistakes within one block, however, if this was the first block of trials, the previous block was administered to ascertain the child's span score. The child's raw score was recorded for each subtest.

Visuospatial working memory

Spatial-simultaneous*:* A grid was presented to the child (firstly a 4 × 3 grid was used, followed by a 4 × 4 grid; all children completed both grid sizes) containing dots. The dots were displayed for 3s before disappearing to leave a blank grid. Immediately following the disappearance of the dots, children were asked to tap on the screen of the laptop being used to indicate where the dots had been. They were instructed that this could be done in any order or pattern. The number of dots per grid ranged from two to eight dots, with blocks of six trials of each number of dots. This reflects the procedure of the WMTB-C. Additionally, the same discontinuation rule was applied. Unlike the subtests of the WMTB-C, a moving-on rule was not employed in this test.

Spatial-sequential, no order*:* The same format of test was used as that for spatial-simultaneous working memory. However, dots were presented one at a time for a period of 1s each on grid sizes 3 × 3 then 4 × 3. All children completed both grid sizes. Again, blocks consisted of six trials, and contained between two and eight dots. Recall in this test was also immediate, with the children being required to tap the screen where the dots had appeared previously. Importantly, children were instructed that they could indicate the location of the dots in any order they wished. This test was designed to determine the role of order during the recall phase in the number of dot positions correctly recalled.

Block recall (Corsi, 1972)*:* The block recall task from the WMTB-C was used to assess spatial-sequential working memory with order. A sequence of blocks is tapped at a rate of one block per second which children must recall in the correct order. Only forwards order recall was required. This test was administered in accordance with the instructions set out by the WMBT-C, as with those used for verbal working memory, hence administration and scoring were as described above.

Mathematics

Access Mathematics Test (AMT)*:* The AMT is a standardised measure of mathematics, available for use with children between the ages of 6 and 12 years. As such it provides a comprehensive profile of how children perform when faced with different aspects of maths. The AMT is aligned to the areas of maths taught on the England and Wales national curriculum, with requirements for children to develop an understanding in the areas of number, measurement, geometry, and statistics, hence providing a valid measure. Questions include those concerning using and applying mathematics (e.g. “tick the two division facts that give the same answer”), counting and understanding number (e.g. “one part of the circle is shaded. How many more parts do you need to shade so exactly one half of the circle is shaded?”), knowing and using number facts (e.g., “what is half of 24?”), calculating (e.g. “complete this calculation and show the remainder: 721 ÷ 2 = __ remainder __”), understanding shape (e.g. “shade in the squares to show the reflection of the shape”), measuring (e.g. “what time does this clock show, in digital form?”), and handling data (e.g.

"the table gives the ages of the members of a golf club. How many members are 55 or older?").

Children were read the instructions set out for the AMT, which included a time limit of 45 minutes, clarification of where to write their answer on the paper, and explanation that workings were allowed on the paper, providing their answer was clearly written in the correct space. Typical classroom test conditions were adopted throughout. Children were permitted to request questions be read aloud to them should they have difficulties so as not to disadvantage those with weaker reading abilities, however, no further explanation of the question, or what was required, was given. No discontinuation rule was employed as children were instructed to complete as many questions as they could, but that questions were also included for children much older than they were so not to worry if they could not complete them all. The total number of test items for this test is 60, with a maximum score of 60.

Data analysis

The R program (R Core Team, 2018) with the "lavaan" library (Rosseel, 2012) was used. Model fit was assessed using various indexes according to the criteria suggested by Hu and Bentler (1999). We considered the chi-square (χ^2), the comparative fit index (*CFI*), the non-normed fit index (*NNFI*), the standardized root mean square residual (*SRMR*), and the root mean square error of approximation (*RMSEA*).

Results

Preliminary analyses

Age (in months) was partialled out of all analyses to remove its influence on the data (see Giofrè, Mammarella, & Cornoldi, 2013 for a similar procedure). Descriptive statistics and correlations are presented in Table 1. There is little variation evident between the raw and covaried correlations, with r values of a similar order in both cases, e.g. $r = .398$ and $r = .399$ for the raw and covaried correlations between spatial-simultaneous 4 × 4 and knowing and using number facts, respectively. Asymmetry and kurtosis were tested on all variables. “Measuring”, a single component of mathematics, was skewed and presented with extremely high values of kurtosis, therefore, this component was removed from further analysis. All other measures had skewness and kurtosis values lower than 1. All the analyses were performed again including measuring and results were extremely similar.

Table 1. *Correlation matrix with raw score correlations below the leading diagonal and covaried scores above, including means and standard deviations for each measure.*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1. Simultaneous 4 x 3	—	.685	.471	.420	.375	.339	.330	.143	.302	.312	.363	.238	.323	.315
2. Simultaneous 4 x 4	.681	—	.408	.425	.409	.297	.335	.091	.283	.251	.399	.285	.300	.296
3. Sequential 3 x 3	.470	.408	—	.550	.300	.266	.254	.086	.212	.230	.275	.195	.165	.197
4. Sequential 4 x 3	.418	.425	.550	—	.312	.185	.259	.093	.207	.190	.227	.212	.145	.224
5. Block recall	.379	.409	.301	.313	—	.238	.264	-.026	.224	.116	.175	.159	.030	.140
6. Counting recall	.343	.298	.267	.185	.241	—	.416	.294	.348	.331	.299	.224	.250	.206
7. Backward digit	.333	.336	.255	.259	.266	.418	—	.253	.287	.313	.342	.240	.122	.131
8. Digit recall	.143	.091	.086	.093	-.025	.295	.253	—	.098	.182	.164	.088	-.050	.112
9. Understanding and app	.322	.278	.213	.204	.234	.351	.289	.099	—	.480	.512	.459	.378	.434
10. Count. and underst.	.318	.251	.231	.191	.121	.334	.315	.183	.483	—	.618	.590	.407	.511
11. Knowing and using	.370	.398	.276	.227	.180	.302	.344	.165	.517	.621	—	.645	.378	.470
12. Calculating	.221	.282	.191	.209	.150	.216	.234	.086	.418	.577	.629	—	.359	.403

13. Understanding shape	.322	.300	.165	.145	.031	.250	.123	-.050	.372	.408	.378	.354	—	.200
14. Handling data	.304	.295	.195	.223	.136	.201	.128	.111	.407	.504	.462	.406	.198	—
M	28.5	2.25	18.84	15.42	21.86	16.47	1.56	26.69	1.77	3.18	2.17	1.66	.9	1.36
SD	5.98	6.93	4.76	4.15	3.55	3.92	2.93	3.14	1.24	1.89	1.43	1.24	.98	1.18
Min.	5	1	7	2	6	6	1	17	0	0	0	0	0	0
Max.	42	39	38	29	32	26	19	47	6	9	6	5	4	5

Note. Correlations greater than .14 are statistically significant at the .05 level. All correlations greater than .18 are significant at the .01 level.

Confirmatory Factor Analysis (CFA)

To confirm the reliability of the structure of the variables, a CFA was conducted. We hypothesized the existence of three separate working memory factors, spatial-simultaneous, spatial-sequential and verbal, and one mathematics factor. The fit of the model was acceptable, $\chi^2(71) = 94.23$, $p = .03$, $RMSEA = .04$, $SRMR = .05$, $CFI = .97$, $NNFI = .97$, and so this model was adopted for the remainder of the analysis (Table 2 and Figure 1). The CFA showed that mathematics is highly correlated with both spatial-simultaneous and verbal working memory, while the correlation with spatial-sequential working memory was moderate. Reliabilities were also calculated from the CFA model using omega, as this is shown to be a more robust measure of reliability at this level (Deng & Chan, 2016; Peters, 2014; Zinbarg, Revelle, Yovel, & Li, 2005; verbal: $\omega = .60$, spatial-simultaneous: $\omega = .81$, spatial-sequential: $\omega = .70$)

Table 2. *Factor loadings, inter-factor and residual correlations for measures included in the model*

	Simultaneous	Sequential	Verbal	Math
Simultaneous				
1. Simultaneous 4 x 3	. 845**			
2. Simultaneous 4 x 4	. 811**			
Sequential				
3. Sequential 3 x 3		.727**		
4. Sequential 4 x 3		.705**		
5. Block recall		.486**		
Verbal				
6. Counting recall			.670**	
7. Backward digit recall			.653**	
8. Digit recall			.370**	
Mathematics				
9. Understanding and applying				.648**
10. Counting and understanding number				.778**
11. Knowing and using number facts				.817**
12. Calculating				.742**

13. Understanding shape				$.491^{**}$
14. Handling data				$.593^{**}$
Inter-factor correlation matrix				
Simultaneous	1			
Sequential	$.758^{**}$	1		
Verb	$.575^{**}$	$.518^{**}$	1	
Math	$.518^{**}$	$.418^{**}$	$.568^{**}$	1

Note.

** $p < .01$.

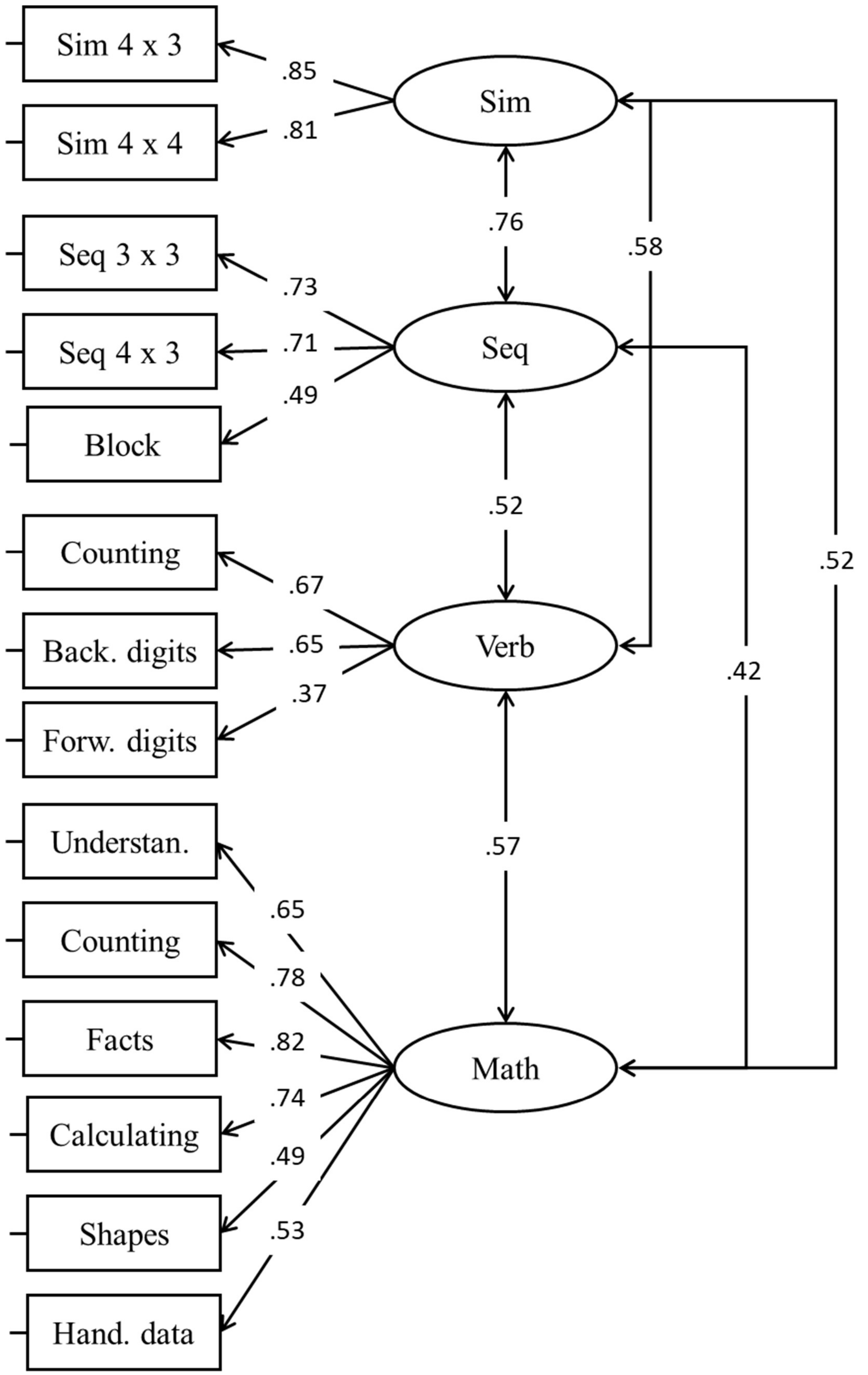


Figure 1. CFA model for spatial-simultaneous, spatial-sequential, verbal and mathematics Coefficients are statistically significant (p < .05).

Variance partitioning

In the final set of analyses, we used variance partitioning to examine the unique and shared portion of the variance of mathematics explained by the spatial-simultaneous, spatial-sequential and verbal factors. A series of regression analyses were conducted to understand the unique and specific contribution of spatial-simultaneous, spatial-sequential, and verbal working memory (see Chuah & Maybery, 1999; Giofrè, Donolato, & Mammarella, 2018, for a similar procedure). As shown in Figure 2, only verbal (10.8%) and spatial-simultaneous (3.4%) factors were explaining a unique portion of the variance of mathematics. Not surprisingly, the larger portion of the variance was shared by the three predictors (15.3%). The total amount of variance accounted for by the model was 37.8%. These findings suggest that a large portion of the explained variance in mathematics is shared, however, some domains, i.e. verbal and spatial-simultaneous working memory, are uniquely predicting mathematics, over and above the effect of the other working memory domains.

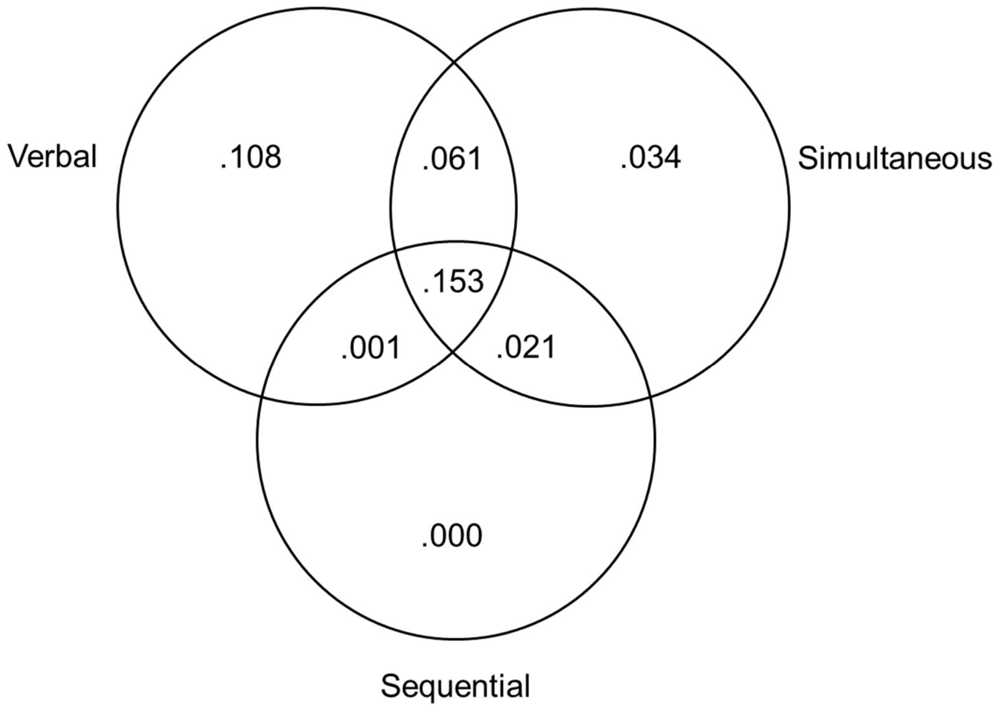


Figure 2. Venn diagram indicating the shared and unique variance explained in mathematics by spatial-simultaneous, spatial-sequential and verbal factors.

Additional analyses

All the analyses were replicated also including "measuring" and the results, not reported, changed very little. Alternative models were tested for working memory. In particular, we tested a single working memory factor, $\chi^2(20) = 71.91$, $p < .001$, *RMSEA* = .12, *SRMR* = .08, *CFI* = .87, *NNFI* = .82, and a three factor solution, $\chi^2(22) = 22.50$, $p = .16$, *RMSEA* = .04, *SRMR* = .05, *CFI* = .99, *NNFI* = .98. These analyses confirm that the fit of the three factor solution, which was adopted in the current paper, was superior when compared to the other two models.

Discussion

This paper aimed to investigate the independent contribution of verbal, spatial-simultaneous, and spatial-sequential working memory to written mathematical performance in 7- and 8-year old children.

From the correlation matrix (Table 1), it is evident that all elements of working memory (besides those correlations between digit recall and spatial-simultaneous 4 × 4, spatial-sequential 3 × 3, spatial-sequential 4 × 3, and block recall) are significantly correlated. All other correlations between each of the measures taken for verbal, spatial-simultaneous, and spatial-sequential working memory were statistically significant, both before and after covarying for age. Results of this nature suggest digit recall may be measuring a different construct to the other measures used to assess working memory, potentially relating to the division of working memory tasks into active and passive tasks (as explained by Passolunghi & Cornoldi, 2008).

In relation to our research question, variance partitioning demonstrates that 15.3% of the variance of maths is shared between the three factors of working memory concerned. The next largest proportion of variance explained is uniquely explained by verbal measures, explaining 10.8% of the variance. This is interpreted as the amount of variance in mathematics accounted for by verbal measures over and above the influence of all other variables measured. This relationship with verbal working memory is consistent with studies suggesting numerically-based verbal tasks are distinguishable from non-numerical verbal tasks and are directly related to children's mathematical performance (see Raghubar, Barnes, & Hecht, 2010 for a review of this literature).

Caution must also be exercised that reading was not measured alongside mathematics, though previous research suggests that the relationship with verbal working memory remains after partialling out reading ability (Wilson & Swanson, 2001; see Simmons, Willis, & Adams, 2012 for a similar argument in relation to elements of mathematics). In the current study, the extent of the impact of this was limited by allowing children to have questions of the mathematics test read aloud to them if they wished. Whilst uptake of this offer was not recorded explicitly, children did make use of the adults present to read the questions for them. In line with previous findings, 37.8% of the variance of mathematical ability was accounted for by verbal and visuospatial measures in total (see Giofrè et al., 2018 and Kyttälä & Lehto, 2008 for similar results).

Interestingly, the results did not show any unique variance explained by spatial-sequential working memory. We had anticipated a larger involvement of spatial-sequential working memory due to the additional active component, however were unsure what the extent of this involvement would be. There are a number of potential explanations for this. The first possible explanation is the ease with which such young children could perform the tasks they were required to do. Whilst unlikely as the sole explanation, as we did not see a floor effect in the data, the results did show positive skew, indicating that the majority of children were performing at the lower end of the scale, therefore, they may have encountered some difficulties with the instructions of the tasks. Note that the spatial-sequential task did not require order during the recall phase, whereas the block recall task did, which may have contributed to the floor effect seen. Secondly, there is evidence that children with high and low mathematical ability are not distinguishable based on their spatial-sequential working memory scores (Bull, Johnston, & Roy, 1999; but see Andersson & Lyxell, 2007; D'Amico & Guarnera, 2005; McLean & Hitch, 1999 for a different argument).

With regard to the contribution made by spatial-simultaneous working memory to mathematics performance, a unique contribution of 3.4% is higher than expected, based on previous literature (e.g., Kyttälä & Lehto, 2008; Swanson & Kim, 2007). This result is a potential by-product of the way in which written mathematics questions are presented in a standard testing procedure. In such a procedure, all information is presented to the child at once and so is available to the child at all times, hence presentation is in line with that of simultaneous working memory measures. Future research could seek to mitigate this effect by presenting a selection of mathematics questions in a sequential format (see Szűcs & Csépe, 2004b, 2004a), to ascertain whether this has any influence over the results gathered. It should be noted that the sample here comprised typically developing children attending mainstream primary schools, none of whom had been identified as exhibiting specific mathematical difficulties. Previous research has identified a relationship between the spatial component of working memory and mathematics (Passolunghi & Mammarella, 2010, 2012), however, this effect has been shown to be stronger in those with a mathematical difficulty (e.g. Mammarella et al., 2018; Peng et al., 2016). As such, it would be reasonable to suggest that a more distinct profile may have resulted from the current study had children with mathematical difficulties been included in the sample.

There are some limitations inherent in this study that it will be necessary to address in future work. Regarding the measures used, verbal measures involved the use of number words, which could feasibly have altered the predictive relationship between verbal working memory and mathematics performance. This is of particular significance in an age group in which one would expect dramatic developmental changes. However, the use of such measures is in line with previous work suggesting a component of working memory responsible for numerical information (as reviewed by Raghubar et al., 2010), hence the results generated are not entirely unexpected.

Continuing on from this, the study concerned only a narrow age group of typically developing children. As such, it is not possible to examine any longitudinal changes relating to age, or to highlight any differences between typical and atypical populations. In fact, from 7 years of age, there is a shift in mnemonic strategy with the emergence of rehearsal in children (Flavell, Beach, & Chinsky, 1966; Susan E Gathercole, 1998; Henry, Messer, Luger-

Klein, & Crane, 2012). It is possible that some children might have used some sort of mnemonic strategy during working memory tasks. For this reason, the relationship between verbal-numeric working memory and mathematic performance may have been underpinned by some sort of a subvocalization process (e.g., rehearsal or other verbal strategies). Overall, it would appear incorrect to assume that children approach the task in the same way (Flavell et al., 1966; Gathercole, 1998). For all these reasons, future studies should be performed to tackle this issue, for example by trying to reduce the use of strategies during working memory tasks.

For tasks that require serial recall there is some suggestion that a common order mechanism is at play (Guérard & Tremblay, 2008). For verbal tasks, it is argued that participants use subvocal rehearsal (e.g., speech-based motor-planning) to maintain the order of to-be-remembered items (e.g., Jones, Hughes, & Macken, 2006). Children involved in this study did appear to use sub-vocal/ vocal rehearsal during the presentation stage, in line with these findings. For visuo-spatial material, the sequence could be maintained via ocular movements (Morey, Mareva, Lelonkiewicz, & Chevalier, 2018; Tremblay et al., 2006). There is some evidence (through similar error patterns; Guérard & Tremblay, 2008) and susceptibility to interference from secondary tasks (Jones, Farrand, Stuart, & Morris, 1995) that the two forms of sequential-order memory have similar underpinnings. It is quite surprising that the verbal sequential task correlated with mathematical performance, but the spatial-sequential task did not. If children are indeed sub-vocally rehearsing, then the relationship to mathematical performance may be attributable to some sort of speech-motor planning (inner speech) that participants engage in when attempting to solve mathematical problems (Rohrkemper, 1986), and not a domain general ordering mechanism (otherwise the spatial-sequential task should have been related to mathematical performance). In a similar vein, several studies indicate that the spatial-sequential working memory component tends to be more strongly related to the mathematical performance in both children with typical development and children with dyscalculia (Mammarella et al., 2018; Passolunghi & Mammarella, 2010, 2012).

One possible explanation for this contradictory finding is that, in the particular mathematical task used, simultaneous processes might take precedent over sequential processing. The mathematical test that we decided to use encompassed several abilities, e.g., geometry and there is some evidence indicating that some simultaneous tasks tend to have a greater contribution as compared to other sequential tasks (see Giofrè, Mammarella, Ronconi, et al., 2013 on this point). This observation is in line with other evidence indicating that the active manipulation of the stimuli tends to be crucial later on in the curriculum, but not in the early stages (see Giofrè, Mammarella, & Cornoldi, 2013 on this point). This is also coherent with the observation that in some tasks, such as fractions, holistic strategies, which require the simultaneous manipulation of visual objects, seem to be very effective as compared to other strategies (e.g., Fabbri, Caviola, Tang, Zorzi, & Butterworth, 2012). Consistently, evidence shows developmental differences indicating that different mathematical training is effective in different age groups (e.g., Caviola, Gerotto, & Mammarella, 2016). Finally, there is some evidence that younger children tend to use less reliable and less efficient strategies prior to a declarative shift in strategy use (see Schneider, 2008 for a review of this), which might have influenced the pattern of results we observed (e.g., Caviola, Mammarella, Pastore, & LeFevre, 2018). For all these reasons, the present findings should be replicated using a more diverse sample including children at different levels of the mathematical curriculum.

The findings from this study have important implications for educational research. An understanding of the elements of working memory that support mathematics development is fundamental for educators aiming to improve children's mathematical attainment. Research is currently trying to exploit this relationship to generate working memory training programmes (e.g., Alloway, 2012; Holmes & Gathercole, 2014). However, at present, randomised controlled trials have not identified evidence of transfer of effects onto academic tasks (e.g., Dunning, Holmes, & Gathercole, 2013), though evidence is mixed (see Morrison & Chein, 2011 for a review of this literature). A recent randomised controlled trial by the Education Endowment Foundation (Wright et al., 2019) identified a non-significant positive influence of working memory training programmes on working memory capacity and mathematics performance when teaching working memory strategies. Caution should be applied when interpreting these results, however, as measures of working memory capacity involved predominantly numerical recall tasks, though children did show additional progress on mathematics measures. It would be of great benefit to educators to understand the predictive nature of working memory for individual components of mathematics as this would enable educators to highlight potential areas of vulnerability in their students. In which case, there is scope for the provision of appropriate aids and alternative methods to be put in place in an attempt to alleviate some of the child's difficulties in that particular area.

In conclusion, this study confirmed a positive relationship between working memory tasks and mathematics attainment. Further verbal-numeric tasks appear to be more predictive of mathematics performance when compared directly to spatial-simultaneous and spatial-sequential tasks, suggesting numerical information is of higher predictive value than visual information when the two are compared directly.

9

Study 1, Paper 2 Introduction

The results of the first empirical paper of this project demonstrated the most sizeable contribution to maths was from verbal working memory, followed by simultaneous visuospatial working memory. There was no unique contribution of sequential visuospatial working memory, despite what may be expected due to the usual predictive powers of more complex tasks, for example those measuring an executive component (e.g. Bull, Johnston, & Roy, 1999; Holmes & Adams, 2006). The contribution made by verbal working memory was highly unexpected, given the literature available suggesting that visuospatial working memory is highly predictive of mathematical attainment in young children (e.g. Fanari, Meloni, & Massidda, 2019; Geary, 2011; Hilbert, Bruckmaier, Binder, Krauss, & Bühner, 2019; Kyttälä & Lehto, 2008; van der Ven, van der Maas, Straatemeier, & Jansen, 2013). Though it is important to consider that the paper considered verbal-numeric stimuli as the verbal stimuli, a relationship that has been found previously to be stronger than that with non-numeric verbal stimuli (as in Raghubar, Barnes, & Hecht, 2010). The misalignment of these results with the literature raised questions regarding whether this relationship was stable for each mathematical component, or whether this could be an influencing factor. As a result, the analysis was re-ran using only verbal and visuospatial factors of working memory (due to the lack of distinct contributions of simultaneous and sequential visuospatial working memory measures), but instead relating these measures to individual components of mathematics. The paper that follows is in press in Educational Psychology and was written in collaboration with Dr. David Giofrè. In this instance, I had designed and conducted the study, Dr. David Giofrè supported the analysis phase. We wrote the paper following questions that arose from the analysis included in the previous paper.

10

Study 1, Paper 2: A distinction between Working Memory Components as Unique Predictors of Mathematical Components in 7-8 Year Old Children

Abstract

Despite evidence for the involvement of working memory in mathematics attainment, the understanding of its components to individual areas of mathematics is somewhat restricted. This study aims to better understand this relationship. 214 year 3 children in the UK were administered tests of verbal and visuospatial working memory, followed by a standardised mathematics test. Confirmatory factor analyses and variance partitioning were then performed on the data to identify the unique variance accounted for by verbal and visuospatial working memory measures for each component of mathematics assessed. Results revealed contrasting patterns between components, with those typically visual components demonstrating a larger proportion of unique variance explained by visuospatial measures. This pattern reveals a level of specificity with regard to the component of working memory engaged depending on the component of mathematics being assessed. Implications for educators and further research are discussed.

Introduction

Mathematics is a very heterogeneous concept, including several different sub-domains. Dating back to the prehistoric times of the hunter-gatherer, the use of mathematics in the forms of number, magnitude, and form can be seen (Boyer & Merzbach, 2011; De Cruz, 2006), however, the term itself is not used until the time of the Greeks. Since its first use by the Ancient Greeks (Boyer & Merzbach, 2011), mathematics has been used as an umbrella term, seemingly accounting for pure arithmetical concepts, as well as other more specific concepts, such as geometry. Boyer and Merzback (2011) describe how the term was coined by the Pythagoreans, and used by those who first began to study mathematics for its own sake. In literature surrounding these times, one can clearly see a distinction between arithmetical and geometrical mathematics.

This sharp distinction between arithmetic and geometry was maintained for several centuries. For example, in the medieval age, arithmetic and geometry were distinguished and, alongside music and astronomy, were included in the so called *qudrivium*, encompassing these four "mathematical" subjects (Grant, 1999). Nowadays, curricula around the world have somewhat abandoned this distinction and we usually refer to mathematics, although a variety of different forms of mathematics exist and seem to be very different from one another.

Different predictors of mathematics performance have been identified but – among several others – working memory, a system for the short-term storage and manipulation of information, has been repeatedly associated with several different mathematic skills. It has been shown that working memory predicts performance on tests of approximate mental addition (Caviola et al., 2016, 2012; Kalaman & Lefevre, 2007; Mammarella, Caviola, Cornoldi, & Lucangeli, 2013), written subtractions (Caviola et al., 2016, 2018), number facts (Steel & Funnell, 2001), multi-digit operations (Heathcote, 1994), magnitude representation (Pelegrina et al., 2015), arithmetical problems (Passolunghi & Siegel, 2001; Passolunghi & Mammarella, 2010), quantitative central conceptual structures (Morra, Bisagno, Caviola, Delfante, & Mammarella, 2019), and geometrical achievement (Giofrè, Mammarella, & Cornoldi, 2014b; Giofrè, Mammarella, Ronconi, et al., 2013). Importantly, working memory is a generic term, for which we also see alternative models.

Several alternative working memory models have been proposed, but the classical tripartite working memory model (Baddeley & Hitch, 1974), which includes a central executive, responsible for controlling resources and monitoring information, and two domain-specific modules for either verbal or visuospatial information, tends to be one of the most well-known (see Baddeley, 2010 for a review). Other accounts postulate the existence of a sharp difference between a working memory factor, which requires cognitive control to a large extent, and a short-term memory factor, which requires less cognitive control (i.e., fewer attentional resources; Kane et al., 2004). Finally, there is a domain-specific factors model, only distinguishing between verbal and visuospatial modalities (Shah & Miyake, 1996). The distinction between verbal and visuospatial working memory, has recently received broader attention and might be of particular importance when considering mathematics as it aligns well with the historical argument that geometry is distinct from arithmetic, dealt with by visuospatial and verbal working memory, respectively, given the nature of the requirements of each.

Only a few studies consider the relationship between verbal and visuospatial working memory in mathematics or in typically developing children. The literature is rife with debate regarding the specific contributions of working memory to academic performance in both typically and atypically developing children (e.g. Alloway & Alloway, 2010; Geary, Hoard, Byrd-Craven, & DeSoto, 2004). Studies have found evidence in support of the stronger influence of visuospatial working memory (e.g., Caviola et al., 2014; Clearman et al., 2017; Holmes et al., 2008; Li & Geary, 2017), however, evidence for the influence of verbal working memory can also be found (e.g., Hitch & McAuley, 1991; Wilson & Swanson, 2001), particularly verbal-numeric working memory (see Raghubar et al., 2010 for a review). Such diverse findings, however, might be attributable to the particular mathematical tasks used in different studies, and it appears plausible to hypothesize that different mathematical subdomains might require verbal and visuospatial working memory resources to a different extent.

The particular relation of the components of working memory to the components of mathematics is as yet a relatively under-researched topic, with much of the literature concerning the relationship between working memory components and mathematics performance as a whole. These particular relationships are not considered in recent meta-analyses, for example by Friso-van den Bos et al. (2013), and Peng et al. (2016). Whilst there have been studies investigating the relationship between working memory and particular elements of mathematics (e.g. arithmetic: Ashkenazi et al., 2013; Caviola et al., 2012; Passolunghi & Cornoldi, 2008, word problem solving: Swanson & Beebe-Frankenberger, 2004; Swanson & Sachse-Lee, 2001; Zheng et al., 2011), this remains an area that requires development. Research into the relationship between working memory and geometry has also received attention (e.g., Giofrè et al., 2014b; Giofrè, Mammarella, & Cornoldi, 2013, as has its relationship with mathematical difficulties (Andersson & Lyxell, 2007; D'Amico & Guarnera, 2005; McLean & Hitch, 1999; Passolunghi & Cornoldi, 2008; Szucs et al., 2013).

A more intricate understanding of the relationships between working memory and the components of mathematics is fundamental before future work can begin on developing interventions targeting children vulnerable to mathematics difficulties. This paper aims to further the debate discussed above by highlighting the differential contributions of components of working memory to different forms of mathematics. In this study, working memory will be divided into verbal and visuospatial components, whilst arithmetic will comprise using and applying mathematics, counting and understanding number, knowing and using number facts, and calculating. Geometry will consist of understanding shape, and handling data in order to encompass tasks that are inherently move visual in nature. These tasks rely heavily on diagrams and mental images of space, hence are intuitively more likely to draw on the visuospatial component of working memory. By assessing each of these areas with regard to the relative contributions of verbal and visuospatial working memory, it will be possible to understand more specifically how mathematics and working memory are related, as well as where to target mathematics interventions for the greatest effect. This analysis is performed on a data set previously analysed in Allen, Giofrè, Higgins and Adams (2020), which demonstrates the strongest unique influence of verbal-numeric working memory on mathematics, followed by spatial-simultaneous working memory (spatial working memory tasks during which all to-be-remembered information is presented simultaneously). This paper seeks to further this understanding to address how the balance of influence identified may be affected by the area of mathematics in question. It is important to note that no overlapping analyses are reported in either paper. We hypothesise that visuospatial working memory will be more influential in geometry due to the inherent visual nature of the tasks, whilst verbal working memory will remain more influential in arithmetic tasks since verbal working memory seems to be involved in tasks requiring fact recall and basic mathematical skills.

Method

Participants

The sample initially included 214 7-8 year old children. Some children were absent during the second administration and so were excluded from the final sample. The final sample included a total of 197 children (95 males and 102 females, M_{age} = 95.99 months, *SD* = 3.63). An opportunity sample of Year 3 pupils in each of the five schools was gathered, using opt-out parental consent to reduce bias in the sample (Krousel-Wood et al., 2006). The study was approved by the School of Education Ethics Committee at the University of Durham.

Parental consent was assumed if opt-out forms were not returned. Children with a diagnosis of a special educational need, including intellectual disability, or neurological or genetic disorder, were not included in the study. Children classed as low functioning or “gifted” are routinely included in typical classes in the UK and were not therefore excluded from our sample.

Procedure

All children were tested individually, in a quiet area of their school. Measures were administered in a randomised order, so as to account for any order effects, however, the size of the grids used in the derived measures of visuospatial working memory were administered in a fixed order (3 × 3 then 4 × 3, and 4 × 3 then 4 × 4, for sequential and simultaneous, respectively). A correlational design was used to explore the relationships between visuospatial working memory and maths performance. Working memory measures were administered as per the administration instructions provided with the WMTB-C, in their original format. Additional visuospatial measures were derived for the purposes of the study, for which administration procedures paralleled those set out for standardised measures, however, were presented using a Windows laptop computer, as opposed to in physical form. The battery of measures used was chosen in order to ensure a fully crossed model for each type of verbal and visuospatial working memory. The mathematics test was presented in paper format, however, children could ask for questions to be read aloud in order to not place children of lower reading ability at a disadvantage.

Measures

Verbal Working Memory

Working memory test battery for children (WMBT-C). Three subtests of the WMTB-C were administered: digit recall (children recall a list of digits presented to them verbally), backwards digit recall (children recall a list of digits presented to them verbally in reverse order), and counting recall (children count aloud the number of dots on a page then recall the list of totals, in the correct order, once all pages in the sequence have been counted). All subtests were administered in accordance with the instructions set out for the WMTB-C, with items presented at a rate of one item per second. Trials were administered in blocks of six trials of the same length. Following four correct trials, testing moved on to the next block. Testing was discontinued following three mistakes within one block, unless this was the first block of trials, in which case the previous block was administered to ascertain the child's span score. A raw score, standard score, and span score was recorded for each child on each subtest.

Visuospatial Working Memory

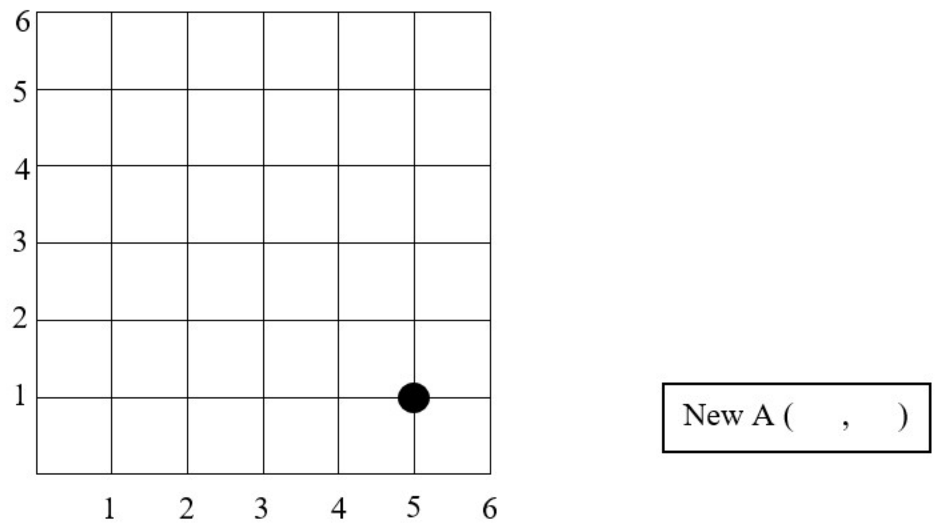
Children were presented with three visuospatial working memory tasks (simultaneous, sequential without order during recall, and sequential with order during recall). For simultaneous and sequential without order tasks, a grid was presented containing dots. The dots were either presented all at the same time (simultaneous) or one at a time (sequential) for 3s and 1s each, respectively. Children were required to observe the positions of the dots and recall these positions following removal of the stimulus. For sequential visuospatial working memory with order, the block recall subtest from the Working Memory Test Battery for Children was employed.

Mathematics

Access mathematics test (AMT). The AMT was employed as a standardised measure of mathematics, available for use with children between the ages of 6 and 12 years. As such it provides a comprehensive profile of how children perform when faced with different aspects of maths. Further, the same measure can be given to older children in order to understand how this relationship with visuospatial working memory may develop over time. The AMT is aligned to the areas of maths taught on the curriculum, hence providing a valid measure whereby performance on the test demonstrates likely performance on Government-prescribed mathematics tests. “Children were read the instructions set out for the AMT, which included a time limit of 45 minutes, clarification of where to write their answer on the paper, and explanation that workings are allowed on the paper, providing their answer is clearly written in the correct space. Typical test conditions were adopted throughout. Children were permitted to request questions be read aloud to them should they have difficulties so as not to disadvantage those with weaker reading abilities, however, no further explanation of the question, or what was required, was given. No discontinuation rule was employed as children were instructed to complete as many questions as they could, but that questions were also included for children much older than they were so not to worry if they could not complete them all” (Allen, Giofrè, Higgins, & Adams, 2020, p. 241). All mathematics testing was carried out after completion of all working memory testing. The two testing phases were on different days for all children. The components of mathematics included were as follows: using and applying mathematics (8 questions), counting and understanding number (12 questions), knowing and using number facts (8 questions), calculating (8 questions), understanding shape (8 questions), and handling data (8 questions) (α = .96 and α = .97 for test forms A and B, respectively).

Questions include those concerning using and applying mathematics (e.g. “circle the two addition facts that give the same answer”), counting and understanding number (e.g. “Circle the number that is nearest in value to 75”), knowing and using number facts (e.g., “what is double 32?”), calculating (e.g. “complete this calculation and show the remainder: 659 ÷ 5 = __ remainder __”), understanding shape (e.g. “a tetrahedron has four corners and four faces. How many edges does a tetrahedron have?” [a picture of a tetrahedron is included for reference] and see figure 1a for a further example), and handling data (see figure 1b for an example). Arithmetic tasks are presented in a variety of ways and ask children to do a number of things from completing calculations to selecting from multiple choice options. Geometric tasks also concern a number of skills, with handling data questions mainly concerning the construction and interpretation of graphs and charts, and understanding shape tasks including tasks encompassing a range of skills such as transformations and properties of shapes.

a) The point A is moved two squares to the left and four squares up. Write the coordinates of the new point A.



The bar chart, from a spreadsheet, shows the number of pets each pupil owns. How many pupils own 3 or more pets?

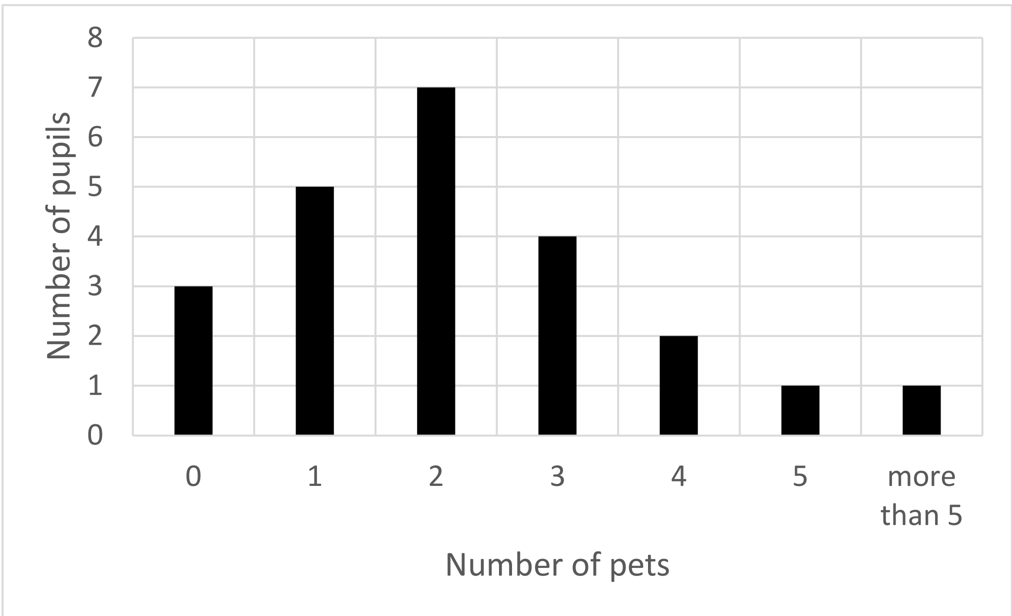


Figure 1. Diagrams showing example questions for understanding shape and handling data subscales of AMT (a and b, respectively).

Data Analysis

The R program (R Core Team, 2018) with the "lavaan" library (Rosseel, 2012) was used. Model fit was assessed using various indexes according to the criteria suggested by Hu and Bentler (1999). We considered the chi-square (χ2), the standardized root mean square residual (SRMR), the root mean square error of approximation (RMSEA), and the comparative fit index (CFI), This data set has been previously analysed in Allen et al. (2020c), however, previous analysis was concerned only with the relationship between verbal and visuospatial working memory and mathematics, but without distinguishing between different mathematic subcomponents. Analyses in the variance partitioning section were performed using the latent correlation matrix for each model. This matrix was used for calculating the R^2 for multiple regressions using the "mat.regress" function available for the "psych" package (Revelle, 2017; see Cohen, Cohen, West, & Aiken, 2013 for the statistical rationale).

Results

Descriptive Statistics

Descriptive statistics and age-covaried correlations are provided in table 1. Age-covaried values were obtained using regressions in which age was entered as a predictor and residuals, controlling for age, were obtained. Age controlled values were then used for all subsequent analyses (see Allen et al., 2020c; Giofrè & Mammarella, 2014; for as similar procedure).

Table 1. *Means and standard deviations, with age -controlled correlations for each measure*.

	1	2	3	4	5	6	7	8	9	10	11	12
1. Simultaneous	1											
2. Sequential	.53*	1										
3. Block recall	.43*	.35*	1									
4. Counting recall	.34*	.26*	.24*	1								
5. Backward digit	.36*	.29*	.26*	.42*	1							
6. Digit recall	.13	.10	-.03	.29*	.25*	1						
7. Understanding and app. math.	.32*	.24*	.22*	.35*	.29*	.10*	1					
8. Count. and under. number.	.30*	.24*	.12	.33*	.31*	.18*	.48*	1				
9. Knowing and using num. facts	.42*	.29*	.18*	.30*	.34*	.16*	.51*	.62*	1			
10. Calculating	.29*	.23*	.16*	.22*	.24*	.09	.46*	.59*	.65*	1		
11. Understanding shape	.34*	.18*	.03	.25*	.12	-.05	.38*	.41*	.38*	.36*	1	
12. Handling data	.33*	.24*	.14*	.21*	.13	.11	.43*	.51*	.47*	.40*	.20*	1
M	48.75	34.26	21.86	16.47	10.56	26.69	1.77	3.18	2.17	1.66	0.9	1.36
SD	11.84	7.84	3.55	3.92	2.93	3.14	1.24	1.89	1.43	1.24	0.98	1.18

Note. * $p < .05$

Confirmatory Factor Analysis (CFA)

In model (CFA00) the factorial structure of working memory, including two components (verbal and visuospatial) was evaluated, results indicated that the fit for this model was adequate (see also Giofrè et al., 2018a for a similar result) and this factor structure has therefore been maintained for subsequent analyses. Successively, we performed a series of CFA analyses, one for each component of mathematics, using the overall scores, and following the general guidelines for SEM with observed indicators (Kline, 2011). Importantly, the fit index of each individual model was good, indicating that a distinction between verbal and visuospatial working memory was adequate. We decided to use CFA because we were mainly interested into the relationship between constructs at the latent level (i.e., verbal vs. visuospatial working memory). Moreover, CFA allows a more precise estimate of the relationship between the construct of interest, reducing problems related to the unreliability of individual predictors (Kline, 2011). Several different models for each individual task were tested in order to obtain baseline estimates (i.e., correlation matrices) to be used in subsequent analyses (see below).

Table 2. *Fit indices for different CFA model*s.

Model	$\chi^2(df)$	*p*	*RMSEA*	*SRMR*	*CFI*	*AIC*
CFA00 WM	8.36(8)	.371	.021	.041	.997	6858
CFA01 UA	10.657(12)	.559	.000	.038	1.000	7458
CFA02 CN	10.608(12)	.563	.000	.038	1.000	7633
CFA03 NF	11.377(12)	.497	.000	.038	1.000	7511
CFA04 Ca	9.103(12)	.694	.000	.036	1.000	7480
CFA05 Sh	27.756(12)	.006	.082	.056	.934	7394
CFA06 HD	11.518(12)	.485	.000	.039	1.000	7464

Note. χ^2= chi-square, *RMSEA*=root mean square error of approximation, *SMSR*=standardized root mean square residuals, *CFI*=comparative fit index, *AIC*=Akaike information criterion. UA=using and applying mathematics, CN=counting and understanding number, NF=knowing and using number facts, Ca=calculating, Sh= understanding shape, HD=handling data.

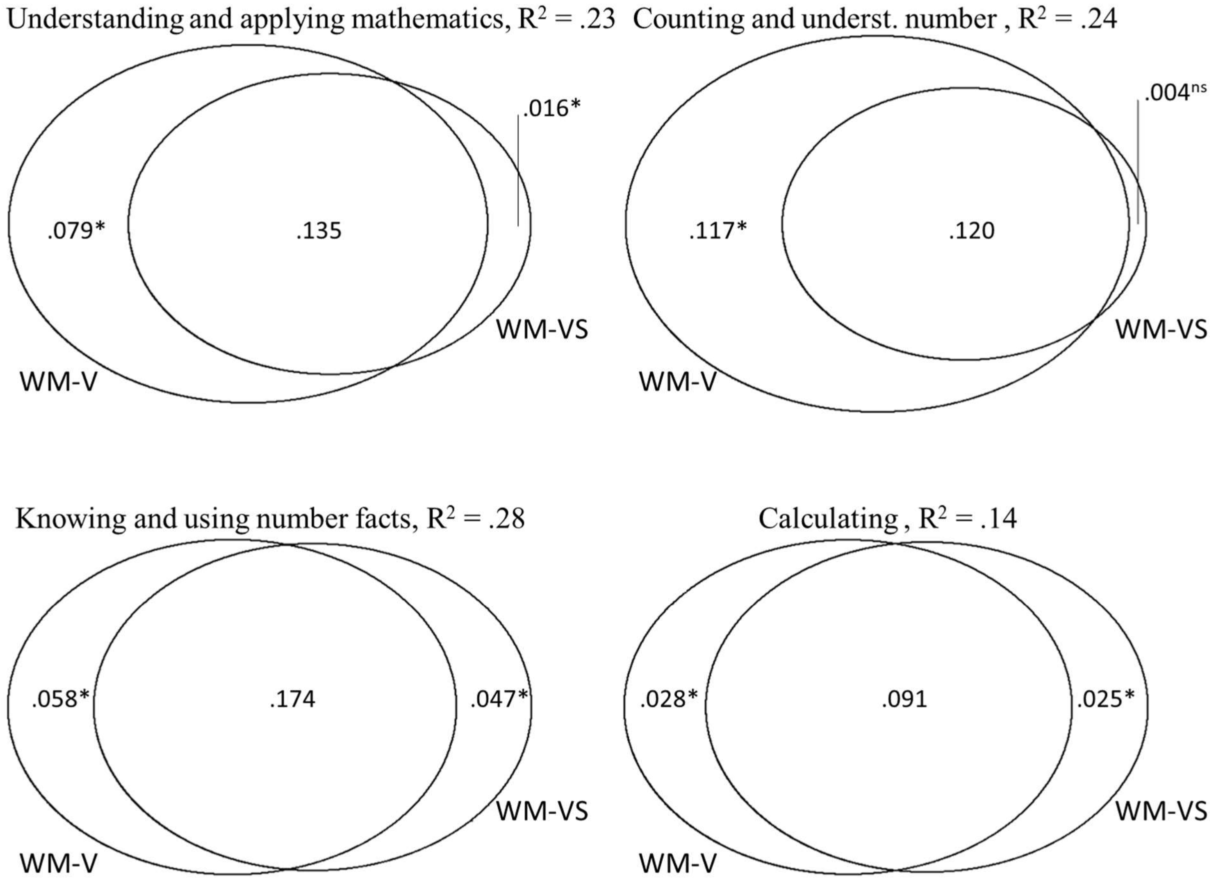
Variance partitioning

In the final set of analyses, starting from the correlation matrices obtained in the CFA, we used variance partitioning to examine the unique and shared portion of the variance of mathematics explained by the verbal and visuospatial factors. A series of regression analyses were conducted (see Chuah & Maybery, 1999; Giofrè et al., 2018a for a similar procedure). To derive the R^2 components for the various tasks, a number of regression analyses must be conducted (Chuah & Maybery, 1999). In this specific case, if verbal working memory is included in the first step (Model 1), while spatial working memory is included in the second step (Model 2), the resulting ΔR^2 corresponds to the unique contribution of spatial working memory over and above the effect of verbal working memory (i.e., R^2 of Model 2 – R^2 of Model 1). Vice versa, if spatial working memory is included in the first step, while verbal working memory is included in the second step, the resulting ΔR^2 corresponds to the specific contribution of spatial working memory over and above the effects of verbal working memory. Finally, the shared variance between verbal and visuospatial working memory can be obtained by subtracting the unique portion of variances uniquely explained by verbal and visuospatial working memory from the overall portion of the variance explained when these indicators are included simultaneously into the equation (i.e., the overall R^2 – unique variance of both verbal and visuospatial working memory).

The variance partitioning analysis is particularly useful for distinguishing shared variance, i.e., the portion of the variance that is common to two or more predictors, and unique variance, i.e., the portion of the variance which is uniquely predicted by one indicator (verbal or visuospatial working memory in this case).

Some mathematics components, i.e., using and applying mathematics, counting and understanding number, are more heavily influenced by verbal working memory (Figure 2), whereas understanding shape and handling data demonstrate a larger visuospatial component (Figure 3).

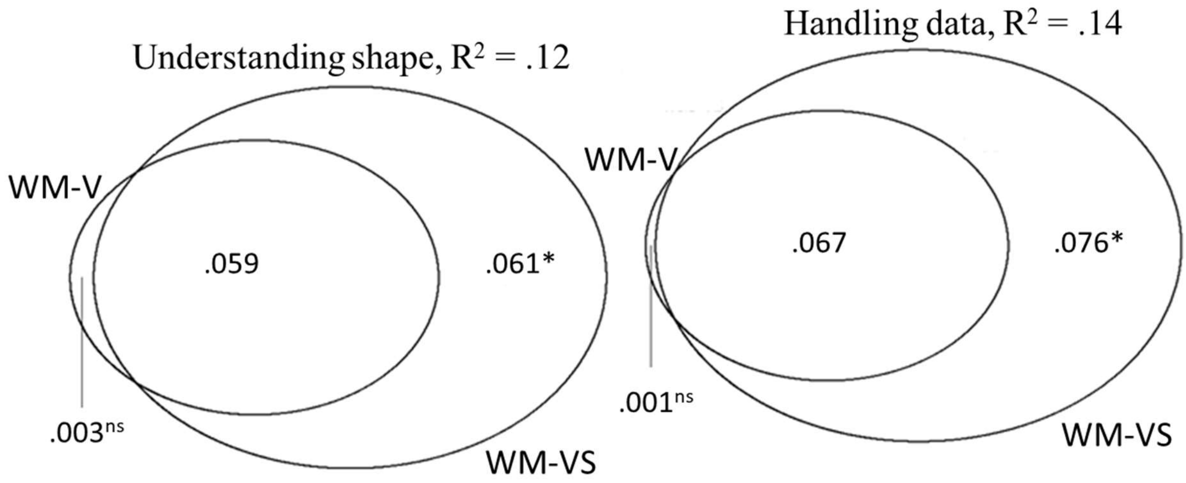
Figure 2. Venn diagram indicating the shared and unique variance explained in using and applying mathematics, counting and understanding number, knowing and using number facts, and calculating by visuospatial and verbal factors. The overall area is proportional within each task, but not across tasks.



* = $p < .05$, calculated using semi-partial correlations.

^{ns} = not statistically significant, calculated using semi-partial correlations.

Figure 3. Venn diagram indicating the shared and unique variance explained in understanding shape, and handling data by visuospatial and verbal factors. The overall area is proportional within each task, but not across tasks.



* = $p < .05$, calculated using semi-partial correlations.

ns = not statistically significant, calculated using semi-partial correlations.

Discussion

The principal aim of this paper was to further understand the individual contribution of verbal and visuospatial working memory to several distinct aspects of mathematic achievement. Previous evidence tends to be limited to the analysis of the overall performance in mathematics while an intricate understanding of the relationships between working memory and the components of mathematics might have important implications for developing interventions targeting children with mathematics difficulties.

From the variance partitioning diagrams, it is evident that the percentage variance accounted for by working memory components varies dependent on the element of mathematics in question. Consistently, the largest percentage is accounted for by shared variance between verbal and visuospatial measures. With regard to using and applying, and counting and understanding number, the next largest percentage is accounted for by verbal measures (7.9% and 11.9%, respectively). This can be interpreted as the amount of variance in these components of mathematics accounted for by verbal-numeric measures over and above the influence of all other variables measured. Such a relationship is in line with previous literature relating verbal-numeric measures to mathematics performance (see Raghubar et al., 2010).

One potential explanation for this relationship emerging for these components is the mental maturation of the children, here accounted for by age. Age appears critical when considering the relationship between visuospatial working memory and mathematics (Holmes & Adams, 2006; Holmes et al., 2008; Li & Geary, 2017) with a stronger relationship demonstrated with younger children. Hence, by the age of the children involved in this study, there may have been a shift to verbal strategies, as suggested by Soltanlou, Pixner and Nuerk (2015). Further, the suggestion of a cyclical relationship between visuospatial working memory and verbal working memory conforms to the assumption that visuospatial working memory relates more strongly to the acquisition of new skills (Andersson, 2008). Consequently, once children reach 7-8 years of age, they may have sufficient experience with the material required for answering questions of this nature that they do not need to rely on visuospatial supports.

When considering knowing and using number facts, and calculating, a different relationship is evident. Whilst verbal-numeric measures technically continue to explain the second greatest portion of unique variance, this difference with visuospatial measures is negligible. One potential explanation for the influence of visuospatial measures on these tasks is the format of the questions. All mathematical questions were presented to children in written format; a format which may inherently engage the visual component (e.g., Wong & Szücs, 2013). This may be particularly potent for measures of calculating as question format has been shown to influence strategy choice (Katz, Bennett, & Berger, 2000). Strategy choice may be more or less “fixed” in different areas of mathematics, depending on children’s familiarity and experience with the component. For areas such as calculating that are taught from an early age and where children have more experience, they may have a greater variety of strategies at their disposal which may be better or worse fit for a question depending on the style of presentation. However, this is speculation in this case as it is beyond the realms of this paper to answer this question. Whilst this may affect written over verbal question presentations, the influence of strategy choice dependent on the layout of written questions (as shown by O’Neil Jr. & Brown, 1998) should be minimal in this study as questions were presented in a variety of ways e.g. multiple choice, open questions. Future research should be mindful of this influence, and could seek to investigate how the layout of the questions themselves may influence method choice, and thus the extent of the involvement of visuospatial working memory (Cragg & Gilmore, 2014).

Perhaps the starkest difference is present between these previous four components of mathematics and the shape and data handling components. In these cases, a shift towards a much larger influence of visuospatial working memory is clear. This shift is as expected, given the visual nature of the tasks, and confirms the heavy involvement of visuospatial working memory in those tasks wherein visual information is paramount to success. Previous work has identified a similar relationship between visuospatial working memory and geometry (e.g., Kyttälä & Lehto, 2008), with complex visuospatial working memory tasks demonstrating predictive power for academic achievement in geometry (Giofrè, Mammarella, Ronconi, et al., 2013), as well as accounting for group differences in performance in geometry between typically developing children and those with a non-verbal learning difficulty (Mammarella, Giofre, Ferrara, & Cornoldi, 2013). Our findings mirror these results, suggesting that further research should be conducted in this area to determine the specific nature of the relationship between visuospatial working memory and shape in order that preventative and/or restorative measures can be devised.

Importantly, evidence of the distinct contributions of elements of working memory to geometry performance has been shown for both typically (Bizzaro, Giofrè, Girelli, & Cornoldi, 2018; Giofrè et al., 2014b; Giofrè, Mammarella, & Cornoldi, 2013) and atypically developing (Mammarella, Giofre, et al., 2013) children, distinct from measures of pure arithmetic. The aforementioned work revealed that academic achievement in geometry was influenced by working memory, with exaggerated differences between typically and atypically developing children in terms of Euclidian geometry as a result of visuospatial working memory performance.

In a meta-analysis focused on working memory updating and its relation with mathematics, it was found that the comparison between verbal and visuospatial working memory subdomains was in fact statistically significant (see Table 4 of the original report), albeit modest in term of magnitude (Friso-van den Bos et al., 2013). Intriguingly, arithmetic, counting and conceptual skills showed lower correlations with visuospatial updating. It is worth noting, however, that Peng and co-authors (2016) in their recent meta-analyses did not find significant differences between verbal and visuospatial working memory regarding their relationship with mathematics. It is also worth mentioning that concerning geometry, these results were based on a very limited number of observations, i.e., seventeen effect sizes for visuospatial working memory and sixteen for verbal working memory, with a very small number of studies overall, thus making it hard to test other moderating effects (e.g., the school Year). Taking these results overall, we can confirm that more research is needed, confirming the importance of evaluating the unique contribution of verbal and visuospatial working memory on each mathematical subdomain.

It is important to note that the relationship identified here, specific to geometry, shows some variation from relationships identified between pure arithmetic components and working memory (e.g., arithmetic: Ashkenazi et al., 2013; Caviola et al., 2012; Passolunghi & Cornoldi, 2008, word problem solving: Swanson & Beebe-Frankenberger, 2004; Swanson & Sachse-Lee, 2001b; Zheng et al., 2011, mathematical difficulties: Andersson & Lyxell, 2007; D'Amico & Guarnera, 2005; McLean & Hitch, 1999; Passolunghi & Cornoldi, 2008; Szucs et al., 2013). Here we see a greater contribution made by verbal working memory (e.g., Wilson & Swanson, 2001), over that contributed by visuospatial working memory (e.g., Caviola et al., 2014; Clearman et al., 2017; Holmes et al., 2008; Li & Geary, 2017), which is not entirely unexpected, given the types of questions associated with assessments of each type of mathematics.

With regard to the alternative models described in the introduction, the findings refute the model by Kane et al. (2004) as we see domain specific contributions despite the inclusion of working memory measures. This model postulates that only short-term memory is domain specific, whilst working memory tasks represent a domain general executive component, though this does not seem to be the case with the results here. In contrast, the results do seem to align with the domain specific findings of the model by Shah and Miyake (1996), however, their measure of verbal working memory involved reading span. Hence, we cannot be sure our findings have not been influenced in some way by the numeric component of the verbal tasks used, which may have increased the strength of the relationships with verbal working memory (see Raghubar, Barnes, & Hecht, 2010 for a review of the influence of verbal-numeric tasks).

In conclusion, this paper highlights a differential relationship between working memory tasks and mathematics attainment, dependent on the component of mathematics in question. Verbal-numeric tasks appear to be more predictive of performance on tasks more closely linked to factual recall and basic mathematical skills. In contrast, we see a stronger influence of visuospatial working memory in components of mathematics with a clear visual element: understanding shape and handling data. This is also in line with evidence indicating that different brain areas are activated in tasks requiring the manipulation of number or space (Arsalidou & Taylor, 2011; Kanayet et al., 2018).

To sum up, mathematics is a very broad term which encompass several different domains, which are probably distinguishable. This should be taken into account in future research, in fact talking about “mathematics” might not make sense, and research should focus on a more in depth understating of different mathematics subdomains. Finally, practitioners working with children with mathematical problems, should try to understand the causes of these problems, trying for example to understand whether or not the impairment is confined to the visual domain (and hence difficulties in tasks requiring the manipulation of visual materials) or in the verbal domain (and hence in tasks which are prevalently requiring the maintenance of words).

11

Study 2 Introduction

Following the findings of Study 1 indicating that the larger portion of the variance of mathematics was accounted for by verbal-numeric working memory, the next step was to investigate whether this relationship remained constant when non-numeric measures of verbal working memory were used, or whether it was specific to verbal-numeric measures. In addition, study 1 included only children in Year 3. As such, there was no way of ascertaining whether the relationship we identified also remained consistent with age or whether there were developmental differences. Previous literature indicates that children undergo a developmental shift at around 7-8 years old (Schneider, 2008), therefore, we would expect to see that the proportional influence of different components of working memory on mathematics would change in line with this. In order to investigate this, we used a year group specific mathematics test, as opposed to the mathematics test in study 1, so that children were able to attempt a greater number of questions for each component of mathematics, regardless of their age, and dependent on their exposure to mathematics teaching. The tests came from a standardised battery designed to be of equal difficulty for each year group, thus making the level of challenge equal between all children involved. With regard to working memory measures, computerised measures were used in this study to reduce human influence during the administration on things such as rate of presentation and intonation. Dual tasks were also included in the new battery to assess true working memory and allow for its differentiation from short term memory in forward recall tasks. The paper that follows is published in the British Journal of Educational Psychology (Allen, K., Giofrè, D., Higgins, S., & Adams, J. (2020). Working memory predictors of mathematics across the middle primary school years. *British Journal of Educational Psychology*, *n/a*(n/a). doi:10.1111/bjep.12339; Appendix K). It was written with collaboration from Dr. David Giofrè, who was involved from the conception of the study, collaborating on study design, analysis and aspects of the writing up of the paper, as the questions had arisen from the analysis of study 1.

12

Study 2: Working memory predictors of mathematics across the middle primary school years

Abstract

Work surrounding the relationship between visuospatial working memory and mathematics performance is gaining significant traction as a result of a focus on improving academic attainment. This study examined the relative contributions of verbal and visuospatial simple and complex working memory measures to mathematics in primary school children aged 6 – 10 years. A sample of 111 children in years 2 to 5 were assessed (*M*age = 100.06 months, *SD* = 14.47). Children were tested individually on all memory measures, followed by a separate mathematics testing session as a class group in the same assessment wave. Results revealed an age-dependent relationship, with a move towards visuospatial influence in older children. Further analyses demonstrated that backward word span and backward matrices contributed unique portions of the variance of mathematics, regardless of the regression model specified. We discuss possible explanations for our preliminary findings in relation to the existing literature alongside their implications for educators and further research.

Introduction

There is an increasing wealth of literature on the relationship between working memory and academic attainment in school-aged children. Working memory can be operationally defined as the capacity to temporarily store and manipulate information, necessary for the completion of complex tasks (Baddeley, 1992). The model of working memory proposed by Baddeley and Hitch (1974) has been developed since its conception to include two slave systems, the visuospatial sketchpad and the phonological loop, responsible for the storage and manipulation of visual and verbal information, respectively (Baddeley, 2000). The visuospatial sketchpad, therefore, supports visuospatial working memory, whilst the phonological loop supports verbal working memory. This working memory model continues to be robust to methodological advances and research findings, and has repeatedly been used in studies conducted with typically developing children (Alloway, Gathercole, & Pickering, 2006; Giofrè, Borella, & Mammarella, 2017; Giofrè, Mammarella, & Cornoldi, 2013).

Several authors suggest that working memory is differentially related to tasks depending on their content, e.g., to specific areas of mathematics (Peng et al., 2016). In particular, numeric verbal working memory seems to be more closely related to number-based mathematical tasks (as in Raghubar, Barnes, & Hecht, 2010), whilst visuospatial working memory shows a stronger relationship with tasks with a clearer visuospatial element, for example geometry (Giofrè, Mammarella, Ronconi, et al., 2013). Allen and Giofrè (2019) demonstrated results of this nature in 7-8 year old children, suggesting one influencing factor on the extent of the influence of working memory on mathematical performance lies in the working memory tasks administered. Similar findings indicating the differential influence of working memory components on mathematics can be found in Andersson and Lyxell (2007), Holmes and Adams (2006), Holmes, Adams and Hamilton (2008), and Nosworthy, Bugden, Archibald, Evans and Ansari (2013).

With regard to mathematics as a whole, results appear largely mixed, seemingly dependent on the measure of working memory adopted. Working memory tasks can be divided into those that measure simple span (whereby participants are required to recall a list of target words/letters/digits/shapes immediately after presentation; Unsworth & Engle, 2007), complex span (whereby participants complete an unrelated processing task alongside the recall task; Unsworth & Engle, 2007), and dual tasks (tasks requiring the active manipulation of the presented stimuli before recall of any kind; McDowell, Whyte, & D'Esposito, 1997). Simple measures of span (sometimes referred to as short-term-memory tasks) do not require an extensive manipulation of the stimuli, while the so called complex span (sometimes referred to as working memory tasks), require some sort of manipulation of the stimulus and generally higher levels of cognitive control (see Engle, 2010 for more information about this distinction). On occasion, those measuring only simple span are considered to be representative of short-term memory processes only (as in Kail & Hall, 2001), however, they are often included in working memory batteries to develop a complete understanding of an individual's memory capacity, particularly when working with young children. Alternative formulations of working memory do not postulate a clear distinction between simple (i.e., short-term-memory) and complex (i.e., working memory) tasks, but advanced the idea that different tasks can be differentiated on a sort of continuum between simple and complex tasks (see Cornoldi & Giofrè, 2014 and Cornoldi & Vecchi, 2004 for a review). It is also noteworthy that very young children might present with some difficulties in dealing with complex tasks, hence simple span tasks could probably provide an insight into their ability to complete tasks of this nature, with fewer task demands.

A recent systematic review by Peng et al. (2016) found a significant positive relationship between working memory and mathematics, however, interestingly, no differences between the contributions of working memory components to mathematics. It is important here to consider that the study compared verbal, numeric, and visuospatial working memory tasks only, using a stringent definition of working memory tasks as only complex span or dual tasks, which are supposed to require more attentional resources (or cognitive load) as compared to simple memory tasks (Engle et al., 1999; Kane et al., 2004). Taking a longitudinal approach is valuable for showing the stability of the existence of a relationship between working memory and mathematics (as suggested by studies proposing a developmental shift during childhood e.g. De Smedt et al., 2009; Van de Weijer-Bergsma, Kroesbergen, & Van Luit, 2015), however, the influence of simple tasks was neglected, which may be especially important for understanding the relationship in younger children (as seen in Holmes et al., 2008).

Allen, Higgins and Adams (2019) addressed this issue with regard to visuospatial working memory, similarly identifying a positive relationship between working memory and mathematics when considering school aged children. This paper further elaborates on the important role of age in the relationship between working memory and mathematics (e.g., Li & Geary, 2013; Soltanlou, Pixner, & Nuerk, 2015; Van de Weijer-Bergsma et al., 2015), highlighting the cumulative nature of knowledge. Hence, mastery is sought in individual aspects of mathematics, rather than in mathematics as a whole. Further, it follows that there is evidence of a declarative shift in strategy use which may influence the components of working memory accessed by mathematics questions (see Schneider, 2008 for a review of this). As such, the age of the participants will be crucial to the expected extent of involvement of each component as the pattern of involvement of working memory components in mathematics varies as a function of age (Friso-van den Bos et al., 2013).

Taking a more holistic approach to the types of working memory tasks used, Friso-van den Bos et al. (2013) conducted a further meta-analysis identifying an association between working memory and mathematics in 4 – 12 year olds. In doing so, they identified an influence of age on the component of working memory with the strongest influence, i.e., visuospatial working memory tasks were more highly correlated in younger children, with verbal working memory becoming more influential as children grew older. Similarly, visuospatial working memory was found to be the dominant deficit in developmental dyscalculia (Mammarella et al., 2018; Szucs et al., 2013). Likewise, a study by McKenzie, Bull and Gray (2003) found comparable results, showing that visuospatial working memory is more strongly related to whole-number calculations in younger children, whilst visuospatial and verbal working memory was related to calculations in older children. Conversely, as previously mentioned, one important influence on the extent of the involvement of working memory tasks may be the individual task demands as the demands of more complex working memory tasks may be quite difficult for younger children. Sweller (1994) suggested that the extent to which working memory components contribute may be a result of the cognitive load of each task, with multistep- and word-problems demanding more working memory resources due to the need for more placeholding and knowledge integration. There is a clear gap in the literature here in exploring the link between task complexity, the age of the children assessed, and the predictive value of such tasks for mathematics performance.

This paper aims to address the gaps in the literature identified above by investigating which components of working memory are more influential in mathematics performance at different ages across the primary school years. The cognitive control required by each individual task has been manipulated. We used simple tasks, i.e., forward span, which required a lower level of attentional control, backward span, which additionally requires children to recall the information in backwards order, and dual tasks, which require children to perform two tasks at the same time and is thought to require higher levels of attentional control. In fact, some working memory models distinguish between a horizontal continuum, for example differentiating between the verbal and visuospatial modalities, and a vertical continuum, in which tasks are differentiated based on different levels of attentional control required (Cornoldi & Vecchi, 2000, 2004). The use of tasks tapping different levels of attentional control and targeting the visuospatial and verbal components was necessary in order to highlight the crucial relationships with mathematics over development. Based on previous work in this area (e.g., Allen, Giofrè, Higgins, & Adams, 2020; Holmes & Adams, 2006) we would expect to see a relatively stable influence of visuospatial working memory, with a shift in the strength of the relationship with verbal working memory. This paper will combine both simple and complex tasks that access the verbal and visuospatial components of working memory in order to provide the basis for developing a more thorough understanding of the influence of such measures on mathematical performance in children aged 6-10 years.

Method

Participants

The sample consisted of 111 6-10-year-old children. All children completed both phases of the administration within the same assessment wave; hence the final sample was of 28 Year 2 (6 -7 years), 26 Year 3 (7 – 8 years), 30 Year 4 (8 – 9 years), and 27 Year 5 (9 – 10 years) children (61 male and 50 female, M_{age} = 100.06 months, *SD* = 14.47). An opportunity sample of the four year groups from one primary school was used, using opt-out parental consent to reduce bias in the sample (Krousel-Wood et al., 2006). The study was approved by the School of Education Ethics Committee at the University of Durham. Parental consent was assumed if no opt-out slip was received. Children with special educational needs, intellectual disabilities, or neurological and genetic disorders were not included in the study.

Design and Procedure

All children were tested individually in a quiet area of their school. The six working memory measures were administered in a randomised order, using counterbalancing to reduce the effects of fatigue and practice. A correlational design was adopted to explore the relationships between working memory and maths performance. All working memory measures were administered in a computerised format, using E-Prime. Two trials of each span length were used, with children completing the whole test to provide a fully saturated measure of their working memory capacity. The mathematics test was presented in paper and pencil format. Children could ask for a question to be read aloud in order to not place children of lower reading ability at a disadvantage. Partial credit score was used for all working memory tasks (as in Giofrè & Mammarella, 2014) whereby participants are credited for all correct responses made in the correct serial position irrespective of whether the full response list was recalled accurately. This measure provides a fully saturated picture of an individual's working memory capacity and allows us to take into account the information from partially accurate lists. The partial credit score is more reliable and accurate as compared to traditional scoring methods, such as absolute credit score (Giofrè & Mammarella, 2014; Unsworth & Engle, 2007).

Measures

The working memory measures used in this paper demonstrated very good psychometric properties and were previously used in other studies with similar populations to the current study (e.g., Giofrè et al., 2017).

Verbal working memory

Three measures of verbal working memory were taken: forward word span, backward word span, and a verbal dual task. Forward and backward word span tasks required children to repeat the list of words they had heard in either forwards or backwards order, respectively (Cronbach`s alpha .71 and .83, respectively). The dual task required children to listen to a number of word lists, all of length 4. Children were required to press the spacebar when they heard the name of an animal, as well as retaining the final word in each list (see Figure, 1 for an example). None of the word lists used contained mathematical and or geometrical words, e.g., rectangle or multiplication. Once they had heard all of the lists for that trial, children were asked to recall the final word from each list in the correct order (alpha = .83). All tasks presented words at a rate of one word every 2 seconds.

For example, if you hear these words:

Cut, crocodile, race, song

Rabbit, sun, street, cloud

???

You should say the words “song” and “cloud”.

Figure 1. Instructions for the verbal dual task.

Visuospatial working memory

Three measures of visuospatial working memory were taken: forward matrices, backward matrices, and a visuospatial dual task, using 4 × 4 grids. Forward (alpha = .72) and backward (alpha = .87) matrices required children to repeat the sequence of black squares they had seen in either forwards or backwards order, respectively. The dual task presented a series of grids with a number of squares coloured grey. In each grid, children saw three black dots one after the other. Children were required to press the spacebar if they saw a dot in a grey square, as well as remembering the position of the last (3rd) dot in each grid. Once they had seen all of the grids for that trial, children were asked to recall the positions of the last dots in the correct order (alpha = .82). All tasks presented stimuli at a rate of one dot/square per 2 seconds.

Mathematics

Head Start Primary Mathematics: The Head Start Primary Mathematics test is a standardised measure of mathematics, providing a year group specific measure of mathematical performance, in line with the objectives of the National Curriculum. Children are required to develop an understanding of number (e.g. Fill in the answer boxes. a) 2 twos are __? b) 11 twos are __?), measurement (e.g. A bag of apples should weigh 22kg. One bag weighs 23.5kg, another weighs 24kg. How much are the bags overweight altogether?), geometry (e.g. Mrs Pott's garden lawn is rectangular. The lawn measures 8m by 9m. What is the perimeter of the lawn?), and statistics (e.g. Look at the bar chart below. How many fewer people like rabbits than hamsters?) according to the National Curriculum in the United Kingdom. The number of questions addressing each of these topic areas was equal across tests. As such, it provides a comprehensive profile of how children perform when faced with questions relating to different aspects of maths. Additionally, each mathematics test is designed to be of equal difficulty, relative to the National Curriculum expectations of each year group. Children were read the instructions for the Head Start Primary Mathematics test before beginning and were allowed a maximum of 1 hour to complete the test. Each test contained 25 questions, thus 60 minutes provided sufficient time for completion. The instructions given included clarification of where to write their answers, explanation that they must follow the individual instructions given for each question (e.g. use a mental/ written method), and that questions may be read to them should they wish. However, no further explanation of the question, or what was required, was given. Typical classroom test conditions were adopted throughout.

Data analytic plan

All analyses were performed using R (R Core Team, 2018). The package "psych" was used to perform regressions (Revelle, 2017) and the package "VennDiagram" for producing Venn diagrams (Chen, 2018). To obtain a more precise picture of the proportion of unique and shared variance among the variables, we utilized variance partitioning methods, which have been successfully used in similar studies (Giofrè et al., 2014a; Unsworth & Engle, 2006). Variance partitioning, also known as commonality analysis, attempts to partition the overall R^2 of a particular criterion variable into portions that are shared and unique to a set of independent predictor variables (Pedhazur, 1997; Unsworth & Engle, 2006).

Results

Preliminary analyses

Descriptive statistics revealed all skewness and kurtosis values were within the bounds of +/- 1, hence parametric tests were used throughout. Correlations (covarying for age) and descriptive statistics are presented in Table 1. We also performed the analyses for each year group. We performed a series of correlations between age in months and each working memory task, and these were not statistically significant.

Table 1. *Correlation, means and standard deviations for each measure.*

	1	2	3	4	5	6	7
1. forward word span	1						
2. backward word span	.514**	1					
3. verbal dual task	.533**	.347**	1				
4. forward matrices	.443**	.391**	.376**	1			
5. backward matrices	.426**	.453**	.330**	.569**	1		
6. visuospatial dual task	.341**	.282**	.427**	.491**	.437**	1	
7. mathematics	.439**+	.613**+	.282**+	.405**+	.533**+	.375**+	1
M	24.1	26.48	13.95	35.32	26.77	13.26	97.61
SD	7.48	6.22	7.12	9.18	12.7	7.77	13.89

Note. False-Discovery Rate (FDR; Benjamini & Hochberg, 1995) correction of the p-values (implemented using the p.adjust function in R) was applied across the six bivariate associations of interest, i.e., between mathematics and each individual working memory task.

** p < .01, one tail.

+ p < .05, one tail, FDR correction.

Analyses on the overall sample

We performed a series of regressions to understand the specific contribution of our predictor to mathematics for the overall group without distinguishing between different age groups.

In the first regression, verbal working memory tasks (forward word span, backward word span, and a verbal dual task) were predicting mathematics. This model was statistically significant, $F(3, 107) = 23.52$, $p < .001$, $R^2 = .40$. In this model, backward word span, $\beta = .53$, 95% CI [.35, .70], was predicting a significant portion of the variance of mathematics while forward word span *β* = .16, 95% CI [-.03, .35], and verbal dual task, *β* = .01, 95% CI [-.16, .19], were not predicting significant portions of the variance of mathematics.

We also performed a similar regression analysis in which visuospatial working memory tasks (i.e., forward matrices, backward matrices, and a visuospatial dual task) were predicting mathematics. This model was statistically significant, $F(3, 107) = 16.39$, $p < .001$, $R^2 = .31$. In this model, backward matrices, *β* = .41, 95% CI [.22, .61], was predicting a significant portion of the variance of mathematics while, forward matrices, *β* = .10, 95% CI [-.11, .30], and visuospatial dual task, *β* = .15, 95% CI [-.04, .33], were not predicting significant portions of the variance of mathematics.

In a final regression, verbal and visuospatial tasks were entered simultaneously as predictors of mathematics. This model was statistically significant, F(6, 104) = 15.81, $p < .001$, $R^2 = .48$. In this model, backward word span, *β* = .43, 95% CI [.26, .60] and backward matrices, *β* = .26, 95% CI [.07, .44] predicted significant portions of the variance of mathematics, while the other predictors were not statistically significant (*β*s < .13, *p*s > .05).

In order to partition the variance, a series of regression analyses was carried out to obtain R^2 values from different combinations of the predictor variables (see Table 2). The results showed that a large portion of the variance was shared (Figure 2). However, both verbal and visuospatial tasks were also predicting portions of unique variance. Variance inflation (VIF) in each individual regression, presented in Table 2, was generally low, i.e., lower than 2.

Table 2. *R^2 values for regression analyses predicting mathematics for various predictor variables*.

Predictor Variables	R^2	F
Visuospatial working memory	.31	16.39
Verbal working memory	.40	23.52
Verbal and visuospatial working memory	.48	15.81

Note. All R^2 values are significant at $p < .001$.

Figure 2. Variance decomposition. WM-V = verbal working memory, WM-VS = visuospatial working memory.

Analyses per age-group

The data were broken down by year group before performing correlational analyses to investigate the strength of the respective relationships between mathematics and each working memory measure, depending on the age of the children (Table 3, see also the Supplementary Materials). Results showed the strongest relationships between mathematics and verbal span backwards in Year 2, backwards matrices and verbal span backwards in Year 3, forward matrices and verbal span backwards in Year 4, and backward matrices and visuospatial dual task in Year 5 (Table 3).

Table 3. *Correlations, means and standard deviations for each measure, distinguished by year.*

Year 2

	1	2	3	4	5	6	7
1. forward word span							
2. backward word span	.346*						
3. verbal dual task	.585**	.296					
4. forward matrices	.124	.068	.189				
5. backward matrices	.325*	.308	.168	.479**			
6. visuospatial dual task	.470**	.047	.422*	.096	.235		
7. mathematics	.274	.738**+	.225	.145	.445**+	.197	
M	19.32	22.00	11.39	28.54	19.00	11.54	93.68
SD	7.53	5.00	7.56	7.63	11.07	5.66	9.85

Year 3

	1	2	3	4	5	6	7
1. forward word span							
2. backward word span	.488**						
3. verbal dual task	.621**	.245					
4. forward matrices	.441*	.405*	.364*				
5. backward matrices	.618**	.514**	.341*	.694**			
6. visuospatial dual task	.417*	.320	.335*	.524**	.589**		
7. mathematics	.559**+	.718**+	.277	.409*+	.682**+	.299	
M	24.96	27.69	13.23	36.31	30.38	10.15	99.85

SD	7.15	6.23	6.86	9.24	12.01	8.09	18.36

Year 4

	1	2	3	4	5	6	7
1. forward word span							
2. backward word span	.364*						
3. verbal dual task	.436**	.330*					
4. forward matrices	.417*	.353*	.411*				
5. backward matrices	.119	.055	.306*	.417*			
6. visuospatial dual task	.010	.203	.203	.500**	.313*		
7. mathematics	.350*+	.614**+	.333*+	.625**+	.428**+	.535**+	
M	25.67	27.97	13.47	38.17	27.90	14.87	99.67
SD	5.71	5.73	5.54	6.86	12.02	7.87	12.24

Year 5

	1	2	3	4	5	6	7
1. forward word span							
2. backward word span	.495**						
3. verbal dual task	.357*	.321					
4. forward matrices	.364*	.169	.357*				
5. backward matrices	.305	.560**	.329*	.424*			
6. visuospatial dual task	.371*	.372*	.608**	.670**	.615**		
7. mathematics	.458**+	.422*+	.348*+	.335*+	.500**+	.500**+	
M	26.48	28.30	17.81	38.26	30.07	16.26	97.26

SD	7.65	5.89	7.22	9.63	12.83	8.05	14.05

Note. False-Discovery Rate (FDR; Benjamini & Hochberg, 1995) correction of the p-values (implemented using the p.adjust function in R) was applied across the six bivariate associations of interest, i.e., between mathematics and each individual working memory task.

* $p < .05$, one tail.

** $p < .01$, one tail.

*** $p < .008$, one tail.

$^{+}$ p < .05, one tail, FDR correction.

Additional Analyses

All the analyses were replicated using a latent modelling approach. In the first step, a CFA was fitted with two factors, verbal and visuospatial working memory. The fit of the model was satisfactory, $\chi^2(8) = 9.90$, $p = .272$, *RMSEA* = .05, *SRMR* = .04, *CFI* = .99, all loadings were statistically significant as well as the correlation between verbal and visuospatial working memory (Figure 3). Alternative models were tested, we fitted a model creating a latent factor including forward span (both verbal and visuospatial tasks), backward span (both verbal and visuospatial tasks), and dual span (both verbal and visuospatial tasks) (Model 2, Figure 4). However, in this model the latent correlation between forward and backward was exceeding one, meaning that these two aspects are very strongly related and should be included in the same factor (Figure 4). For this reason, the distinction between two factors, verbal and visuospatial working memory, was maintained. We therefore decided to go further and test an additional model including a third factor, i.e., mathematics. To do so, we first created an individual score for each topic and the resulting scores were used to create a latent variable, i.e., mathematics. The fit of the model (Model 3) was adequate, $\chi^2(62) = 107.09$, $p < .001$, *RMSEA* = .081, *SRMR* = .068, *CFI* = .914 (Model 3; Figure, 5). The correlation matrix obtained in Model 3 was used to perform variance partitioning (see Giofrè et al., 2014 for a similar procedure). Results were very similar to those obtained using the observed variables, with visuospatial working memory only explaining 7.7% of the variance, while verbal working memory was explaining about 15.6% of the total variance, while most of the variance was shared between these two variables, i.e., 23.8%. These results, although based on a relatively small sample size, confirm the results obtained above using observed variables, rather than latent factors. The VIF for the model including both verbal and visuospatial working memory was lower than 2.

The effect of age is moderate and statistically significant when the overall sample is considered (*r*s > .245, *p*s < .01) (see also Logie & Pearson, 1997; Huizinga, Dolan, & van der Molen, 2006).Therefore, analyses on the overall sample were repeated controlling for age in months, i.e., for each individual working memory task, we performed a regression including age as a predictor and each individual working memory task as the responding variable, residuals were then saved and used in subsequent analyses. Results varied very little, significant paths remained significant and changed very little in terms of magnitude. The effect of age was not statistically significant within each year of assessment, and when performing a series of partial correlations controlling for age in months in each year of assessment results were very similar in terms of magnitude and changed very little. As for the variance partitioning, results were very similar: 22.2% of the variance was shared, 18.5% was explained by verbal working memory, and 9.1% was explained by visuospatial working memory.

There is a disagreement in the current literature on whether the performance on the forward and backward version of the span (both verbal and visuospatial) is similar or different, with children recalling fewer items in the backward version of the span, which should require more attentional resources (see Donolato, Giofrè, & Mammarella, 2017 for a review). We therefore decided to compare the performance in the forward and backward visuospatial and verbal span in the current sample using a series of repeated measures ANOVAs. As for the visuospatial span, we found a statistically significant difference between the two version of the span, $F(1, 100) = 72.07$, $p < .001$, *Cohen's d* = 0.77, with children recalling more items in the forward version of the span than in the backward. The opposite pattern was found for the verbal span, $F(1, 100) = 13.41$, $p < .001$, *Cohen's d* = -0.34, with children recalling more items in the backward version of the span, but these differences were somewhat smaller in terms of the effect size compared to the visuospatial working memory ones.

The correlation between Age and Grade was very high (r = .94), meaning it is very hard to distinguish between the two. However, it could be argued that the shown pattern of links between working memory and mathematics might reflect the test content rather than be evidence of a developmental shift. We originally decided to use grades rather than the actual age of the children in the analysis as this reflects the mathematics they have experienced. To address this issue, however, we performed a series of meta-analyses dividing the sample into different ages, rather than grades, and comparing the correlations within the age groups, i.e. within seven-year-olds, eight-year-olds, etc. In this analysis the effect of Age as a moderator was investigated. The analytic strategy adopted in this meta-analysis, followed the guidelines proposed by Borenstein, Hedges, Higgins and Rothstein (2009), and by Schwarzer, Carpenter and Rücker (2015). R was used in all the analyses (R Core Team, 2018) and meta-analyses were performed using "metafor" (Viechtbauer, 2010). All values were transformed into the Fisher's Z scale before computing the meta-analysis (see Borenstein et al., 2009 for more details). Estimated coefficients were obtained using the "restricted maximum likelihood" method, which is set by default in the "metafor" package functions. Age did not reach statistical significance as a moderator of the relation between math performance with forward word span, backward word span, verbal dual task, forward matrices, and backward matrices (*p*s > .302). However, as far as the dual task spatial is concerned, we found a statistically significant effect of Age, B = .016, p = .0318 (Figure 6). Showing that pattern of relationship tends to be higher with older children.

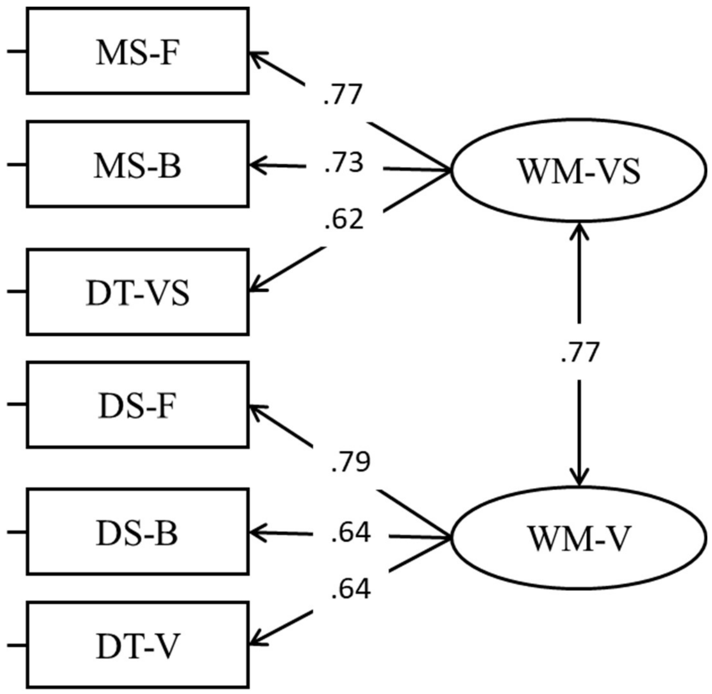


Figure 3. Loadings and correlations for Model 1.

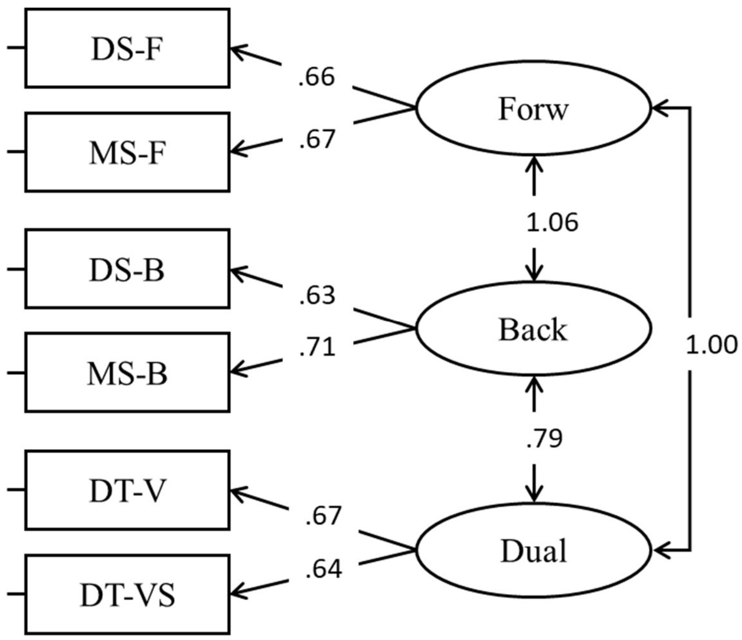


Figure 4. Loadings and correlations for Model 2. Correlations of 1 or higher indicate that factors are not empirically distinguishable.

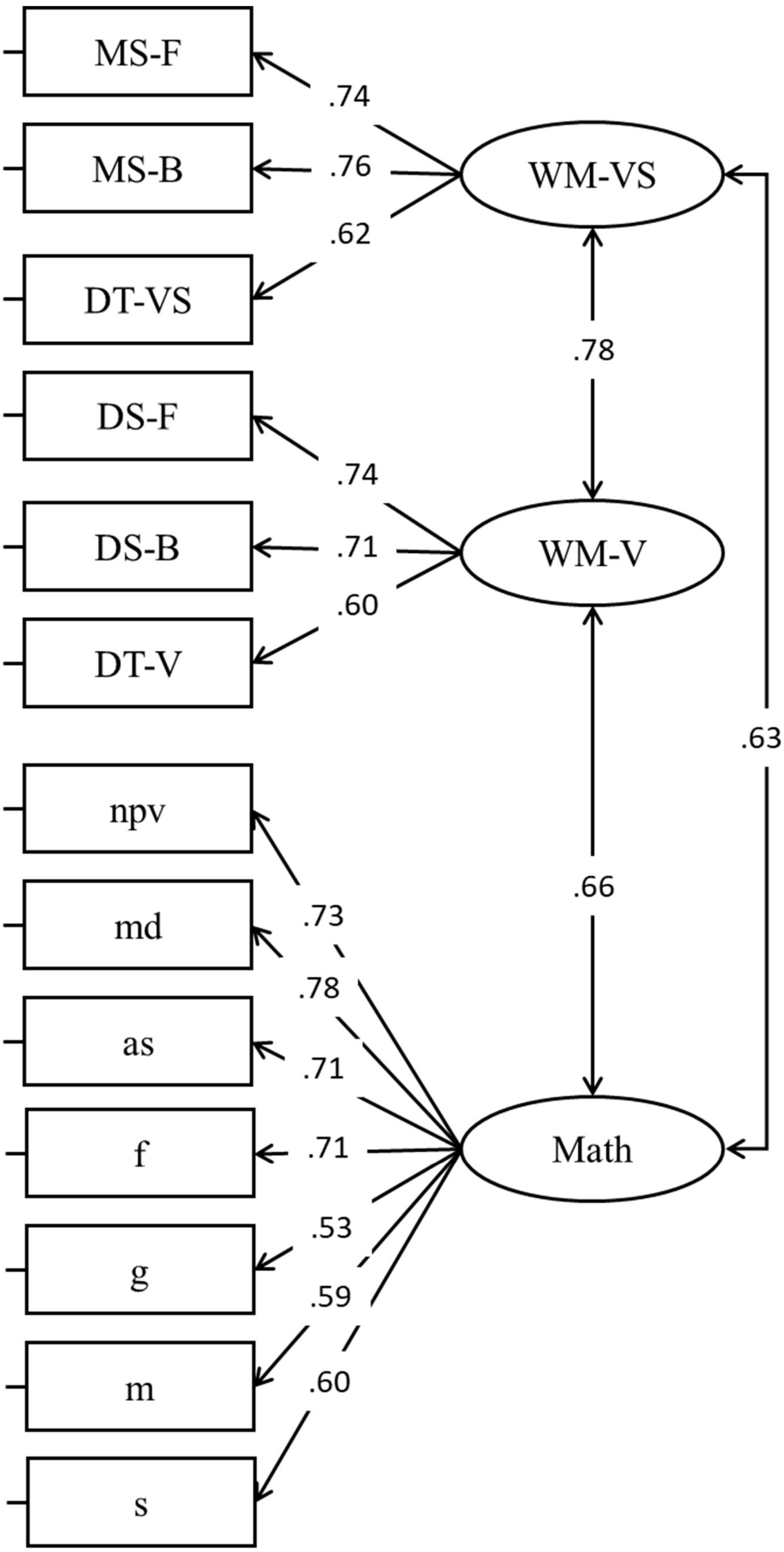


Figure 5. Loadings and correlations for Model 2. Correlations of 1 or higher indicate that factors are not empirically distinguishable.

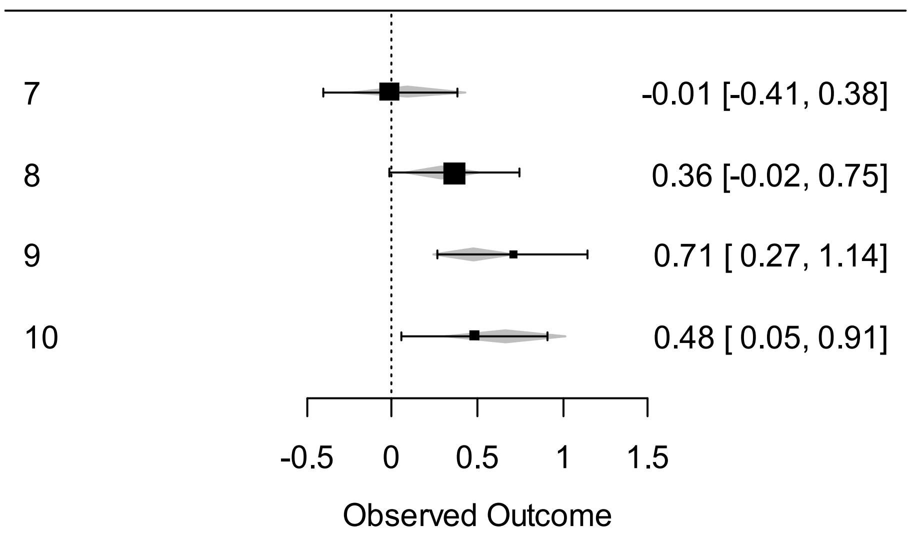


Figure 6. Fisher's Z transformed correlations for the relationship between mathematics and dual task spatial as a function of age. Error bars represent 95% confidence intervals while grey diamonds represent predicted effects at each age.

Discussion

This paper aimed to investigate the independent contributions of visuospatial and verbal working memory to mathematical performance in 6 - 10-year-old children (Years 2 – 5 in the United Kingdom).

From the correlation analyses (Table 1), we can see that all elements of working memory are correlated both with each other and with mathematics, with mathematics being most strongly correlated with backward word span and backward matrices. These correlations were determined after covarying for age, indicating that the relationship with these working memory components is relatively stable. These results suggest that there is an element of the task inherent in backwards tasks that lend them to being more highly related to mathematics than forwards tasks. This is potentially the need for more active processing than required for forwards tasks, which are often viewed as requiring fewer attentional resources (e.g. Passolunghi & Cornoldi, 2008; for a description of tasks whereby active tasks require an additional level of manipulation see Vecchi & Cornoldi, 1999; Vecchi, Richardson,

& Cavallini, 2005). Further, backwards tasks facilitate rehearsal (Conway et al., 2005), with the stimuli being repeated sub-vocally (for verbal tasks; Baddeley, 1992; Smith, Jonides, Marshuetz, & Koeppe, 1998) or in terms of ocular movements (for visuospatial tasks; Tremblay, Saint-Aubin, & Jalbert, 2006) a number of times in order for the participant to accurately reverse the order, producing the final item each time (i.e. n, n – 1, n – 2, etc.) until the entire list has been reversed. This would in itself improve recall if children were afforded the opportunity to rehearse the sequences.

Looking more specifically at the aim of the research, results show that 47.4% of the variance of mathematics performance can be explained by the working memory measures used. Variance partitioning demonstrates that this can be broken down into 15.9% unique variance explained by verbal working memory, 7.7% unique variance explained by visuospatial working memory, and 23.8% shared variance between verbal and visuospatial working memory. Unique variance is interpreted as the amount of variance explained by measures of that component of working memory, over and above the influence of all other variables measured e.g. that of verbal working memory is the variance accounted for by verbal measures over and above the influence of all other measures taken. Here we see the greatest proportion of unique variance accounted for by verbal measures, followed by visuospatial measures. The largest proportion of variance accounted for by the model is that of shared variance between measures that cannot be attributed solely to verbal or visuospatial measures. This pattern of results is consistent with the findings of (Allen et al., 2020b), but suggests that the influence of verbal-numeric tasks may not be as great as suggested by Raghubar et al. (2010) in their review, beyond the influence of non-numeric verbal tasks, as non-numeric verbal tasks also account for a portion of unique variance in mathematics of a similar magnitude (Allen et al., 2020b). Allen et al. (2020) used a numeric span, making the findings difficult to generalise to verbal working memory as a whole. The magnitude of the influence of working memory measures remains stable compared to other studies in the field who identify a similar percentage of variance accounted for (see Giofrè, Donolato, & Mammarella, 2018 and Kyttälä & Lehto, 2008 for similar results). The amount of shared variance evident in the model may also be related to the previously mentioned strategy choices made by the children (Hecht, 2002; Keeler & Swanson, 2001), for example visuospatial tasks where children recode the locations as words may draw on both sources of working memory. Without recording strategy choice it is impossible to take this explanation beyond speculation, leaving the potential for future research to investigate whether strategy choice influences the amount of shared variance explained in the models. It is worth noting, however, that this conclusion is very tentative since this study did not differentiate between numeric and non-numeric verbal tasks, and hence the percentages of explained variance are compared across studies that use different methods and tasks.

A further aim of the study was to assess whether the working memory contributions to mathematics changed with the age of the child as a result of a developmental shift around this time (e.g., De Smedt et al., 2009). We chose to divide the children based on their year group for the analysis because this would be the most appropriate way of controlling for the level of schooling of each child, and thus their exposure to different mathematical concepts. Introducing bias in this is lessened as the year group-based mathematics tests each contained an equal number of questions relating to the areas outlined by the National Curriculum (number, place, and value [n=4]; multiplication and division [n=4]; addition and subtraction [n=4]; fractions, decimals, and percentages [n=4]; geometry [n=3]; measurement [n=3]; statistics [n=3]) and were designed to test the specific requirements of the National Curriculum for each year group. Hence, two children, both aged seven, but in years two and three, would each receive a mathematics assessment relating to the topics they had been taught to that point. This helps to establish an understanding of learning in relation to teaching, which could not be accurately compared otherwise. Chronological age comparisons would be less appropriate in this situation given the different topics each child has been taught, based on their month of birth. Drawing a cut-off for age between January and December or September and August is an arbitrary designation, particularly when the difference in age may be of less than a month. Therefore, using the academic calendar in this situation is more appropriate as the children assigned to each year group will have experienced the same level of schooling.

Interestingly, verbal span backwards showed the strongest correlation with mathematics from Year 2 to Year 4; only in Year 5 was this correlation overtaken by visuospatial tasks. This is contrary to our initial prediction and to previous work that has identified a strong influence of visuospatial working memory in younger children (e.g., Bull, Espy, & Wiebe, 2008; Holmes & Adams, 2006). One possible explanation for this is that all information is presented as words in a written mathematics test, potentially confounded by research showing the presence of reading difficulties relating to difficulties in areas of mathematics (Gersten, Jordan, & Flojo, 2005). Whilst we attempted to mediate the influence of reading ability by offering children the opportunity to have questions read aloud, the only way to negate this influence completely would be to present all questions only orally, providing written copies of diagrams where necessary (see Booth & Thomas, 1999 for an example of this method). However, this method of presentation would still draw heavily on verbal working memory as children would be required to recall larger amounts of verbal information, for which they only had the opportunity to hear once. As regards formal mathematical testing, written presentation is the preferred method in schools, hence understanding the influence of working memory components when problems are presented in this way will be more beneficial in the long term to the development of interventions, as this will develop an understanding of a child's ability to work in the manner in which they will be tested.

Following on, considering the later influence of dual tasks on mathematics, as children get older, the type of questions they are asked to complete become more demanding, often containing multiple steps within one question. Inherent in this is the requirement to process larger volumes of information simultaneously for each question, and this requires attentional control resources and higher cognitive processing to a greater extent (see Giofrè, Mammarella, & Cornoldi, 2013 for a similar argument). As such, it follows that a working memory task that requires an additional level of manipulation is likely to be more representative of the kinds of processes required for mathematics questions written for older children. Geary, Hoard, Byrd-Craven, Nugent and Numtee (2007) identified a number of working memory mediators for both simple and complex mathematics questions relating to this idea. Further, visuospatial tasks became more strongly correlated with mathematics from Year 3 onwards. The relationship with backwards matrices in Years 3 and 5 fits with the assumption that a more active task aligns more readily with demanding mathematics tasks, which require more than simple repetition to complete (Friso-van den Bos et al., 2013; Giofrè, Mammarella, & Cornoldi, 2013). This finding is consistent with the observation that highly controlled working memory processes tend to be more strongly related to higher cognitive abilities both in typically developing children (Cornoldi, Orsini, Cianci, Giofrè, & Pezzuti, 2013) and in particular populations (Cornoldi, Giofrè, Calgaro, & Stupiggia, 2013). Further, questions presented to older children often contain additional information in the form of tables and diagrams, which would serve to engage the visuospatial components of working memory more readily than the simpler presentations (see Reuhkala, 2001 for a similar argument).

There are limitations intrinsic to the study design that further research should seek to address, alongside the above suggestion regarding strategy choice. The main difficulty when administering the tests was the selection of dual tasks used with such young children. Children in Year 2 (6-7 years) struggled considerably to comprehend the dual tasks, and as such did not manage to successfully complete the secondary task alongside the primary task in most cases. In future, it would be beneficial to develop a more easily comprehensible dual task that younger children are able to understand sufficiently well as to be able to complete both elements in order to establish an accurate measure of their capabilities in these kinds of tasks. Further, a sample only containing typically developing children is unable to highlight any potential differences between typical and atypical populations. Given the known differences in working memory capacity between typical and atypical populations (e.g., Swanson, 1993), it would be informative to collect data demonstrating the longitudinal differences in the contributions of working memory to mathematics in these populations to understand whether these are entirely distinct from typical populations or whether they exhibit any overlap. From such work, it would be possible to further understand whether those with mathematical difficulties demonstrate a pattern of developmental delay, or a distinct cognitive profile to typically developing children.

We used regressions in order to control for shared variance between variables for the year-to-year assessment (Loehlin & Beaujean, 2016). However, more sophisticated methods are also available (e.g., Gaussian Graphical Model), which allow accessing a conditional dependence/independence of several variables within one model in each group (Costantini et al., 2015; Epskamp & Fried, 2018). In the present report, we decided not to use these methods because of the relatively small sample size, but these methods could successfully be used in future studies with larger samples. Due to the limited sample size within we decided not to statistically compare correlations coming from independent samples. Future studies with larger sample sizes should be performed to address this issue, for example using more sophisticated techniques such as Multigroup Confirmatory Factor Analyses or Multigroup Structural Equation Modelling. Finally, future studies should try to compute separated scores for different mathematical subareas, such as, number and geometry. We decided not to perform such analyses here because this would have increased the number of statistical comparisons, and this was not ideal with the current sample size. Given that we used the same working memory measures for all ages, but year-specific maths tasks, it could be argued that the different tasks on a same topic still differed on allocation of visuospatial and verbal working memory resources for task completion and caused the reported correlational patterns, e.g., Year 2 statistics might have differed from Year 5 statistics and required different cognitive effort in comparison to tasks for other year groups. Future studies should test this hypothesis.

The findings presented above have important implications for educational research as well as for educators in terms of developing interventions to improve mathematical attainment in those with poor mathematical attainment. In order to improve mathematical attainment for those children who demonstrate mathematical difficulties, first a comprehensive understanding of the ways in which working memory supports mathematical development is necessary. The results of this study indicate some potential longitudinal changes in the influence of working memory components on mathematical attainment, however, also suggest stable elements of influence. Although this paper is only able to identify age-related differences in the contributions of working memory components to mathematical performance at a single point in time in children of different ages, it suggests that future work may seek to identify whether these changes also occur within individuals over development. In a similar vein, we decided to use grade-specific math assessments in different grades, which is the standard in studies investigating mathematics. However, there is no guarantee on how the math outcomes of one grade are comparable to those of another grade. Whilst the exact amount of unique variance accounted for by verbal and visuospatial working memory components at each of the age groups assessed here remains unknown, due to the constraints of sample size, educators would benefit greatly from understanding how these influences change over the primary schools years. In doing so, interventions can be more specifically targeted to provide children with alternative methods that may be better able to support their mathematical development by employing different elements of their working memory.

In conclusion, these preliminary results echo those derived from our previous data (Allen et al., 2020b) that verbal and visuospatial working memory both make unique contributions to mathematical attainment. Further, verbal tasks continue to account for a larger proportion of unique variance, despite the largest proportion being shared variance between both verbal and visuospatial working memory. Finally, this work demonstrated a change in the strength of the correlations between measures with age, showing that more complex visuospatial tasks become more highly correlated with mathematics as children become older, whilst verbal task correlations remain relatively stable.

13

Study 3 Introduction

Study 2 revealed nine children in Year 3 and 4 who performed 1SD or more below age expectations in the given mathematics test and were categorised, for the purposes of the following study, as atypically developing performers in mathematics. From their results a further question arose which was whether it was possible to identify this same group of children through a reduced battery of the working memory measures used in study 2. A reduced battery will be used as not all measures used previously were significantly related to mathematics. We also sought to identify any fundamental cognitive differences between low achievers and the remainder of the sample in order to begin to understand whether there are any consistent cognitive correlates of poor mathematics. As previously mentioned, children were tested on a subset of the working memory measures shown to be the most highly related to mathematics in study 2, with the addition of speed of processing and number sense measures, and a proxy measure for general intelligence (g factor). The principle aim of this study was to determine whether identifying children who are likely to be poor performers in mathematics is feasible from understanding their cognitive profile. This paper was written by myself, with the guidance of Prof. S. Higgins and Dr. J. Adams.

14

Study 3: Using cognitive predictors to predict poor mathematics performance in 7 and 8-year-old children: a feasibility study

Abstract

There is a large body of literature highlighting the role of working memory in mathematics attainment, however, it is unclear whether this relationship remains stable when other cognitive predictors are included. This study aims to investigate this by simultaneously measuring working memory, speed of processing, g, and number sense. 28 children were assessed on all measures. Results show that none of the regression models generated were significant, with no suggestions of fundamental differences between children who performed poorly in mathematics and their peers. Further, analysis at the individual level revealed a great deal of heterogeneity in the cognitive profiles of children showing a cause for concern in mathematics. These results are discussed in relation to existing literature. We conclude that the approach is feasible as long as the chosen measures thoroughly explore the child's cognitive profile. Consideration is also given to the prescription of remediation strategies for poor mathematical performance.

Introduction

A range of cognitive measure have been demonstrated to be influential in children's mathematical attainment (e.g. Fuchs et al., 2006; Geary, 2011). Working memory is one of these measures (see Raghubar, Barnes, & Hecht, 2010 for a review), with a host of studies confirming its predictive power in a number of mathematical skills, such as arithmetic (e.g. Logie, Gilhooly, & Wynn, 1994; Rasmussen & Bisanz, 2005), problem solving (e.g. Swanson & Beebe-Frankenberger, 2004; Swanson, Jerman, & Zheng, 2008), and geometry (e.g. Giofrè, Mammarella, & Cornoldi, 2014; Giofrè, Mammarella, Ronconi, & Cornoldi, 2013). Other measures that have demonstrate these abilities also include speed of processing (e.g. Geary, 2011), general intelligence (e.g. Aluja-Fabregat, Colom, Abad, & Juan-Espinosa, 2000), and number sense (e.g. Nosworthy, Bugden, Archibald, Evans, & Ansari, 2013). Allen, Giofrè, Higgins and Adams (2020a) demonstrated a predictive relationship between two backwards span working memory tasks (backward word span and backward matrices) and mathematics across a group of children aged 6-10, hence we aim here to identify whether this relationship remains following the inclusion of a series of control measures also known to be related to mathematics. The study was designed as a feasibility study (Bowen et al., 2009) to evaluate the practicality and likely utility of exploring this relationship with a view to designing a diagnostic assessment for a group of children whose performance raised concerns.

Speed of processing

Speed of processing relates to the length of time it takes for an individual to process information, which is known to improve as children get older (Fry & Hale, 1996; Kail, 1991; see Fry & Hale, 2000 for a review). Processing speed relates to measures of short-term (Rebecca Bull & Johnston, 1997) and working memory (Fry & Hale, 2000), with the suggestion being that a slower rate of processing leads to an increased level of decay from memory (Case et al., 1982; Towse et al., 1998). Speed of processing has been demonstrated on a number of occasions to predict mathematical ability in children (Bull & Johnston, 1997; Floyd, Evans, & McGrew, 2003; Geary, 2011), with Rohde & Thompson (2007) showing the specificity of this predictive value to mathematics. It is noteworthy that Hoard, Geary, Byrd-Craven and Nugent (2008) identified no significant improvements for mathematically precocious children with regard to speed of processing, suggesting the influences of this cognitive ability may be limited beyond a certain point. It suggests a deficit related to poor speed of processing, but no associated benefit of improved speed of processing beyond that of typical mathematical achievement. Importantly, Berg (2008) demonstrated that speed of processing did not negate the influence of working memory on arithmetic, suggesting that the current study may still show a significant predictive relationship between working memory and mathematics, despite the inclusion of additional cognitive measures.

General intelligence (g)

General intelligence, often referred to as g, is known to be highly related to working memory (Colom, Abad, Rebollo, & Chun Shih, 2005), with some studies even suggesting the two are isomorphic, given the high correlations reported (Kyllonen & Christal, 1990). Results from Conway, Cowan, Bunting, Therriault and Minkoff (2002) highlight this relationship further, demonstrating that working memory capacity can be used to predict g. However, Colom et al. (2005) suggest that when the storage component of working memory is factored out, the isomorphism of the two constructs disappears. This is echoed by Conway, Kane and Engle (2003), who highlight the differential executive component inherent in working memory tasks. This finding indicates that the underlying constructs are different, although the mechanisms by which g functions are not yet fully operationalised.

There are a host of studies that seek to understand how g relates to academic achievement. General cognitive ability has been shown to add to the prediction of academic achievement (Rohde & Thompson, 2007), with fluid intelligence contributing specifically to mathematical attainment, showing a somewhat mediating effect on visuospatial working memory (Kyttälä & Lehto, 2008). More specifically, fluid intelligence appears related to early abilities in arithmetic (Hornung et al., 2014). From such results, it is conceivable that g may mediate the influence of working memory in this group of children, given the specific relationships identified between g and mathematics.

Number sense

Finally, number sense has also demonstrated a relationship with later mathematical performance (Jordan, Glutting, & Ramineni, 2010; Nosworthy et al., 2013), though its investigation is not without controversy (Berch, 2005). Number sense describes an individual's ability to make rapid judgements regarding the quantity of different sets (Feigenson et al., 2013). It has been used to explain behaviours as far back as the hunter gatherer who was able to identify where was 'more' for hunting purposes (De Cruz, 2006). More recently, and in terms of academic achievement, number sense has demonstrated predictive power for mathematics over a time span of a number of years (Mazzocco et al., 2011; Starr et al., 2013), particularly when children were required to compare numerals (Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013). This relationship was also demonstrated to be independent of measures of g (De Smedt et al., 2009). Therefore, it is reasonable to suggests that this measure either contributes to explaining additional variance in mathematics performance or could serve to mediate the relationship identified with working memory. This will be dependent on whether the underlying mechanisms for completing number sense and working memory measures are the same of different.

This study aimed to assess whether the relationship highlighted by Allen et al. (2020a) is sufficiently stable and unique as to remain when additional cognitive control measures are included. We included measures of speed of processing, g, and number sense to assess whether the inclusion of these measures negates the influence of working memory on mathematics. This study was conducted as an exploratory study to further understand the nature of the predictive relationship between cognitive measures and mathematics and to assess the feasibility of the approach.

Method

Participants

The sample consisted of 28 seven- to eight-year-old children, from Years 3 and 4 at a local primary school. The children were identified based on their mathematics score for Allen et al. (2020a), and had been selected due to falling in the lowest scoring half of their classes on the mathematics test used in Allen et al. (2020a). The final sample was of 13 males and 15 females, with a mean age of 105.5 months. With regard to the mathematics scores from the previous study, this sample included a minimum mathematics score of 12% and a maximum of 62% (mean = 42.86%, SD = 15.19%). Of these children, nine were identified as performing 1SD below age expectations and so were identified as the *concern group*. A systematic sample was used from two year groups used in the Allen et al. (2020a) study, using opt-out parental consent to avoid introducing bias into the sample (Krousel-Wood et al., 2006). The study received ethical clearance from the University of Durham School of Education Ethics Committee. No children with formal diagnoses of special educational needs, intellectual disabilities, or neurological or genetic disorders were included in the final sample. This was to provide as typical an overall sample as was possible and practical.

Design & Procedure

Testing was completed on an individual basis in a quiet area of the child's school. The cognitive measures included were administered in a counterbalanced order to reduce the influence of fatigue and practice. For the whole sample analysis, a correlational design was adopted to explore the relationships between the cognitive measures and mathematics. For the subsample analysis, mean difference testing was performed to highlight any fundamental differences in the cognitive profiles of children selected into the concern group, relative to their peers. Measures of working memory were administered in a computerised format, using E-Prime, whereas measures of speed of processing, g, and number sense were administered in a paper and pencil or verbal format in accordance with the administration instructions of the WISC-IV and the Numeracy Screener. In the case of working memory measures, two trials of each span length were administered, with no cut off criterion to ensure a fully saturated measure of a child's working memory capacity. Partial credit score was used for these tasks (as detailed in Allen et al., 2020a) to account for partially accurate recall of some items. The structure of all other measures, including scoring and cut off criteria, was predetermined by the standardised measures used.

Measures

The measures used in this study demonstrate very good psychometric properties, having been used in other studies with similar population (e.g. Giofrè, Borella, & Mammarella, 2017) in the case of the working memory measures, or having been standardised on large populations that are reflective of the population in question.

Working memory

Allen et al. (2020a) identified backward word span, and backward matrices as significant predictors of mathematical performance in this age group. Backward word span required children to repeat a list of words they had heard in backwards order (Cronbach's alpha = .83). Backward matrices required children to repeat the sequence of black squares they had seen in backwards order (Cronbach's alpha = .87). Both tasks presented stimuli at a rate of one item per 2 seconds, and Cronbach's alpha scores are taken from Allen et al. (2020a) where they were calculated on a larger sample, which included the children in this sample.

g

Two measures from the WISC-IV were taken to form a measure of g: matrix reasoning and vocabulary. Matrix reasoning presents children with a series of tiles and requires them to select which tile from the selection at the bottom of the page completes the pattern above. These patterns are relatively easy to complete to begin with and get progressively harder as the test progresses. The vocabulary subtest presents children with a series of words that they are required to provide the definition for. Again, this test begins with relatively easy objects/words to define, with words becoming more difficult to define as the test progresses. Psychometric properties for these measures can be found in the WISC-IV manual.

Speed of processing

Two further measures from the WISC-IV were administered to measure speed of processing: coding and symbol search. Both tests are time limited to two minutes for the child to complete as many items as they are able to. Coding presents children with a series of boxes with a number in the top and a blank space below. A key is presented above the stimulus material to indicate which symbol should be written in the blank space depending on the number. Children are instructed to complete as many of these as they are able to in the time limit, whilst also working as carefully as they can. Symbol search presents children with a target symbol in one column and a selection of symbols in an adjacent column. For each trial, the child must identify if the target symbol is present in the selection of symbols in the adjacent column. Again, children should complete as many items as possible in the time limit, whilst being careful not to make mistakes.

Number sense

To measure number sense, the Numeracy Screener (Nosworthy et al., 2013) was used. The screener presents children with pairs of boxes. Children must decide which of the boxes in each pair contains more; more dots or the number that is larger in value. This measure is timed to allow children one minute per part (numerals or dots) and they are instructed to work as quickly, but as accurately, as they can. Details of the development and standardisation procedure can be found in Nosworthy et al. (2013).

Data analysis

All data were analysed using R (R Core Team, 2018). The main packages used for the analysis were “lavaan” (Rosseel, 2012) and “psych” (Revelle, 2017), along with “ggplot2” (Wickham, 2016).

Results

Preliminary analyses

In order to remove its influence on the data, age in months was partialled out before beginning any analysis. Table 1 presents the descriptive statistics and correlations for all of the measures taken. All skewness and kurtosis values fell within reasonable bounds of ± 1.5, therefore, parametric tests were used throughout the analysis. Given the size of the sample, it is important to note that all of the following analyses are tentative and preliminary in nature. The study is designed to examine the feasibility of using these measures to predict mathematics only, not to draw any firm conclusions from the data.

Table 1. *Correlations, means, and standard deviations for each measure taken.*

		1	2	3	4	5	6	7	8	9
1	Symbolic NS									
2	Non-symbolic NS	.696**								
3	Symbol search	.326	.218							
4	Vocab	.014	.006	.091						
5	Matrix	.301	.350	-.084	.263					
6	Coding	.201	.058	.514**	-.021	-.203				
7	Backward matrix	-.150	-.027	.148	.437*	.052	.326			
8	Backward word	.000	.071	.129	.511**	.075	.216	.472*		
9	Maths	.190	.230	.259	.228	.407*	.157	.342	.294	
	Mean	35.43	35.54	21.39	28.21	14.68	38.82	30.75	25.64	21.50
	SD	6.49	5.06	5.76	5.31	4.23	9.60	12.09	5.66	7.58

** $p < .001$ *$p < .05$

Analysis on the whole sample

We began the whole sample analysis by examining the correlations table (Table 1). Measures of number sense, speed of processing, and working memory correlate significantly with each other ($p < .05$ in all cases) as would be expected for measures tapping the same underlying component. Backward word span and backward matrices also correlated with vocabulary measures ($p < .01$ and $p < .05$, respectively). Interestingly, matrix reasoning was the only measure to correlate significantly with mathematics ($r = .407$, $p < .05$).

We conducted a series of regressions to understand the contributions of our predictors on mathematics. In the first model all measures were entered individually to predict mathematics. This model was not statistically significant, $F(8,19) = 1.274$, $p = .313$, $R^2 = .349$. The second model included only the measures regarded as control measures. Again, this model was non-significant, $F(6,121) = 1.363$, $p = .275$, $R^2 = .280$. Finally, the third model included only the working memory measures. This final model was also non-significant, $F(2,25) = 2.028$, $p = .153$, $R^2 = .140$.

Scatterplots of the data, Figure 1, suggest that the relationships between the cognitive measures and mathematics are not linear in every case, hence may explain some of the findings from the regression models.

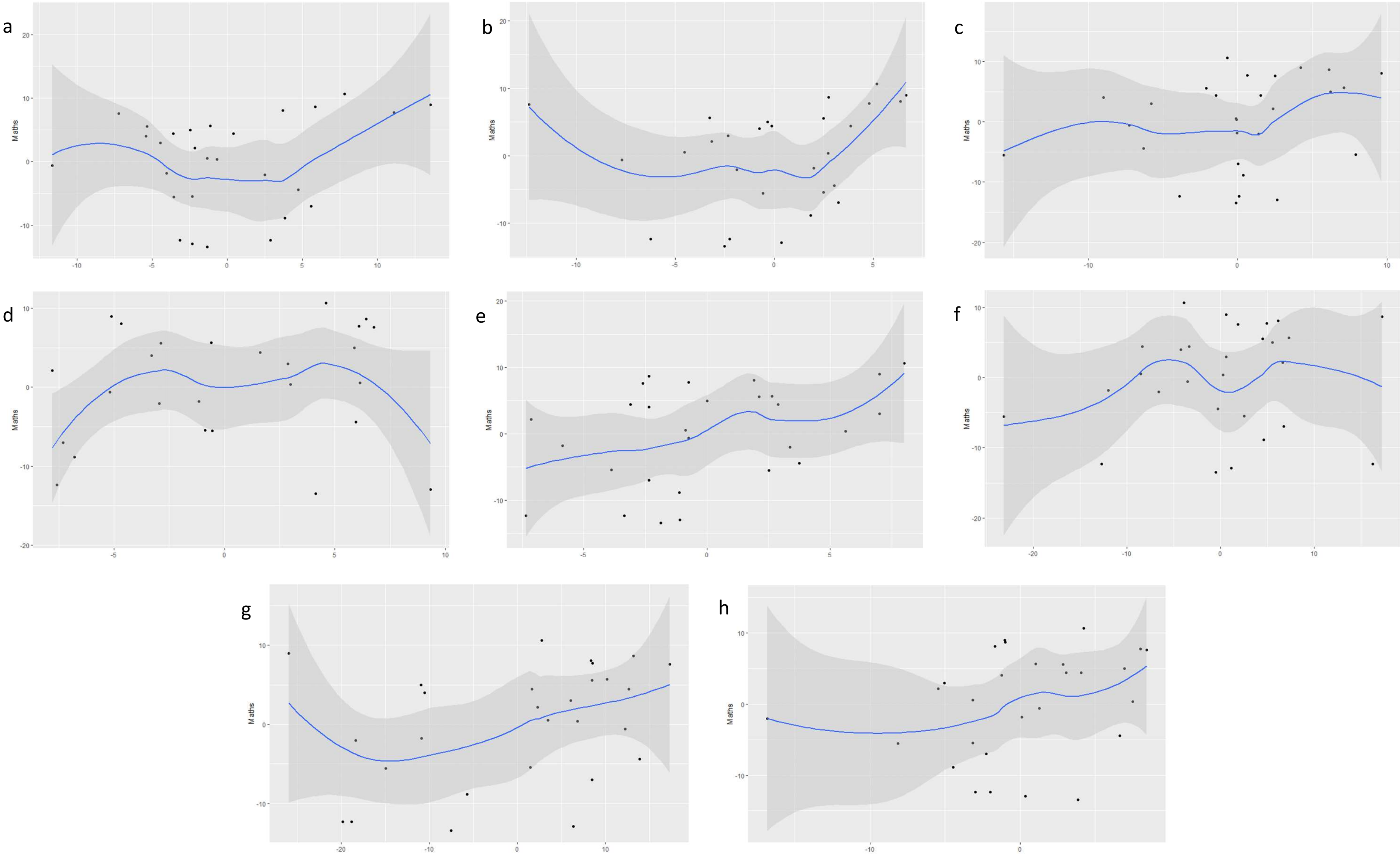


Figure 1. Scatterplots showing the distribution of each variable against mathematics scores. a) Symbolic NS b) Non-symbolic NS c) Symbol search d) Vocabulary e) Matrix reasoning f) Coding g) Backward matrices h) Backward word span

Subgroup analysis

The nine children that had been highlighted from previous analyses (Allen et al., 2020a) as the concern group were gender matched with children in the remaining sample to create a control group. These two groups were then compared against each other.

We ran a series of exploratory logistic regressions to see whether it was possible to predict group membership from the cognitive measures taken. The first model included each cognitive predictor individually to predict group membership. The model did not contain any significant predictors, *AIC* = 37.344. The second model included number sense, speed of processing, and g as composite scores, and each working memory measure individually. As with the first model, there were no significant predictors, *AIC* = 33.079. The third model included only the composite control measures to assess whether these measures alone could differentiate between the groups. The model contained no significant predictors, *AIC* = 32.024. We then combined the speed of processing and number sense measures into a single measure, as they all rely on timely processing, and included this along with g and working memory measures. Again, this model did not result in any significant predictors, *AIC* = 32.156. The final model considered only verbal (symbolic NS, vocabulary, backward word span) and visuospatial (non-symbolic NS, symbol search, matrix reasoning, coding, backward matrices) predictors. Similarly, there were no significant predictors returned, *AIC* = 29.18.

We then ran a series of ANOVAs to ascertain whether the data showed any significant differences between the groups on any of the cognitive measures included. The results of the ANOVAs are included in table 2, however, showed no fundamental underlying differences between the two groups in this sample.

Table 2. *Mean and SDs for each measure for each group, F and p values for ANOVAs calculated between concern and control group, and Cohen's d effect sizes.*

	Concern group Mean (SD)	Control group Mean (SD)	*F*(16)	*p*	*d*
Symbolic NS	.494 (3.696)	-.526 (4.017)	.314	.583	0.26
Non-symbolic NS	-.059 (3.168)	.789 (3.587)	.282	.603	-0.25
Symbol search	-1.671 (6.560)	1.276 (5.646)	1.043	.332	-0.48
Vocabulary	-1.256 (6.532)	-.153 (4.067)	.185	.673	-0.20
Matrix reasoning	-1.642 (3.319)	-.503 (3.167)	.555	.467	-0.35
Coding	-.587 (11.375)	.066 (9.345)	.018	.896	-0.06
Backward matrices	-4.095 (12.338)	1.813 (11.861)	1.072	.316	-0.49
Backward word span	-1.347 (4.425)	-1.867 (5.977)	.044	.836	0.10

Individual profile analysis

Before beginning this section of analysis, all scores were standardised to centre on a mean of 0. For the working memory measures, means and SDs from the previous, larger data set were used (see Allen et al., 2020a). The mean z scores for each measure, along with their respective 95% CIs, can be found in table 3. Note, number sense is not included in this section as the only available standard measure of performance is percentile rank, normed only on children in the Canadian education system in Ontario. Interestingly, matrix reasoning is the only measure for this sample where both the upper and lower CI boundaries lie below age norms. The children included in this section of analysis are those who were included in the concern group previously.

Table 3. *Standardised means and 95% CIs for this sample.*

	Mean	Lower CI	Upper CI
Symbol search	.107	-.172	.386
Vocabulary	.071	-.259	.402
Matrix reasoning	-.726	-1.053	-.400
Coding	-.083	-.434	.268
Backward matrices	.141	-.249	.532
Backward word span	-.364	-.736	.009

From these means and CIs, we assessed whether the individual normed score for each child fell within, above or below the 95% CI for the group. As can be seen from table 4, there were no obvious patterns to the data. In various combinations, children showed evidence of deficits in all included areas (speed of processing, g, and working memory). Some children showed no clear evidence of deficit in any of the areas measured. Profile plots can be seen in figure 2 to highlight the heterogeneity seen in the cognitive profiles of a sample of children who are all performing at least 1SD below age expectations in mathematics. Though caution should be applied in interpreting limited conclusions from feasibility data, the data shows that 6/9 children show some evidence of some kind of working memory difficulty, 5/9 show some evidence of some kind of speed of processing deficit, and 4/9 show some evidence of some kind of deficit in g.

Table 4. *Children's cognitive profiles showing whether their performance on each task falls above, within, or below the 95% CI for this sample. Also included for reference is their maths score.*

Symbol	-1.33	-1.00	-.33	.00	.66	-.67	1.00	-.33	-.33
search	Below	Below	Below	Within	Above	Below	Above	Below	Below
Vocab	.33	-.33	-.33	2.00	-.66	.67	-.33	.33	-1.66
	Within	Below	Below	Above	Below	Above	Below	Within	Below
Matrix	.00	-2.00	-1.33	-1.00	-1.00	.00	-1.33	-1.00	-1.33
reasoning	Above	Below	Below	Within	Within	Above	Below	Within	Below
Coding	-1.33	2.00	-1.33	.00	1.33	-.33	-.33	-.67	-.33
	Below	Above	Below	Within	Above	Within	Within	Below	Within
Backward	-1.28	-1.61	-1.7	.47	.72	1.42	.42	-.32	-.16
matrices	Below	Below	Below	Within	Above	Above	Within	Below	Within
Backward	-1.88	-.91	-1.07	-.59	-.75	.88	-.69	.53	-.87
word span	Below	Below	Below	Within	Below	Above	Within	Above	Below
Maths	14	7	7	6	14	18	18	10	15

Note. Each column denotes an individual child.

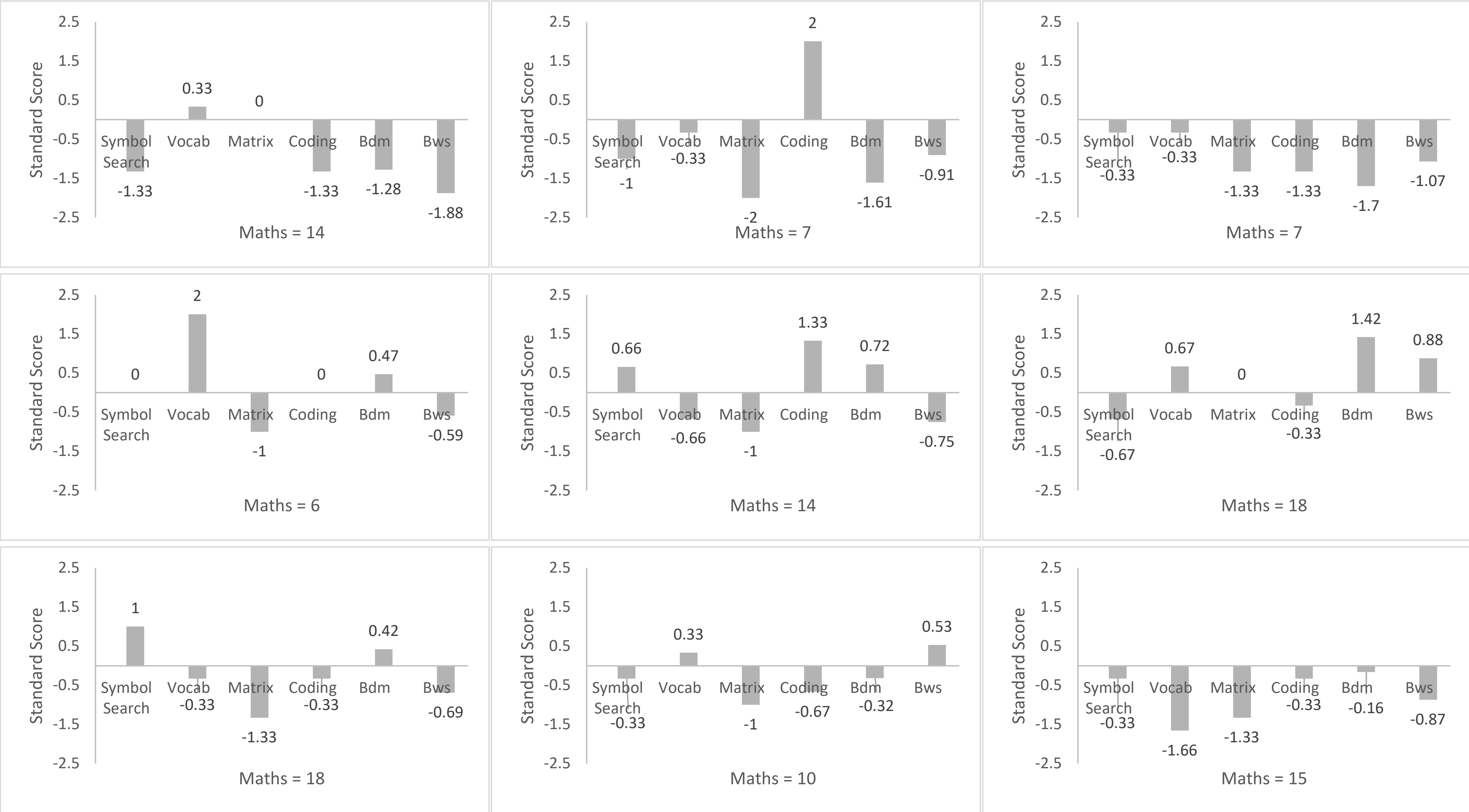


Figure 2. Profile plots showing the scores for each individual in the concern group on the cognitive measures taken and their mathematics score.

Discussion

Before thorough discussion of the findings of this study, it must again be emphasised that all analyses are done on a preliminary, exploratory basis to identify any potential differences in the cognitive profiles of children who performed poorly on a mathematics test conducted earlier in the same academic year. In addition, we remind readers that these findings cannot be taken to be robust with such a small sample, however, they do give an important insight into the cognitive profiles of the children of interest. Further, they highlight the finer nuances that practitioners must be aware of when working with children who are struggling with mathematics.

As previously mentioned, the aims of this study were to conduct a feasibility analysis to explore whether the inclusion of cognitive measures known to correlate highly with mathematics (Anobile et al., 2018; Fuchs et al., 2006; Geary, 2011; Odic et al., 2016) negated the predictive powers of working memory tasks as identified in Allen et al. (2020a), or whether these relationships were maintained. In short, we hoped to ascertain whether this reduced battery of working memory measures continues to highlight the same children as a cause for concern as the battery used in Allen et al. (2020a) when used alongside control measures. By doing so, we hope to give an indication of whether these relationships are robust when a fuller range of measures are included in the model. Secondly, we aimed to assess whether there were any significant differences in performance on cognitive measures between the concern group and gender-matched controls. As such, we hoped to identify the potential for any underlying fundamental differences between the groups that could explain the differences in performance on the mathematics test. Finally, we sought to explore the individual profiles of the children in the concern group to understand whether any patterns in performance were present.

Whole group analysis

Beginning with the analysis of the whole sample, the individual measures that form the composite measures of working memory, speed of processing, and number sense correlate significantly. However, we do not see the same correlation between the individual measures for the composite score of g. No significant correlation is seen here, suggesting that the vocabulary and matrix reasoning measures may be assessing different underlying elements of cognition. This is not unreasonable, given vocabulary relates to verbal intelligence (Bornstein & Haynes, 1998; Goldstein, Allen, & Fleming, 1982; Regard, Strauss, & Knapp, 1982) and matrix reasoning relates to visuospatial intelligence (Haavisto & Lehto, 2005), however, it would be expected that these measures would correlate with each other should they be measuring the same underlying construct: intelligence.

Matrix reasoning is the only variable to correlate significantly with mathematics, with all other correlations remaining relatively low. This is curious, given previous research identifying strong and significant correlations between mathematics and working memory (Andersson & Lyxell, 2007; De Smedt et al., 2009; Meyer et al., 2010; Swanson & Kim, 2007), speed of processing (Rebecca Bull & Johnston, 1997; Lambert & Spinath, 2017; Vanbinst et al., 2018), number sense (Halberda, Mazzocco, & Feigenson, 2008; Jordan, Kaplan, Locuniak, & Ramineni, 2007; Starr et al., 2013; see Gilmore et al., 2013 for an alternative argument), and other measures of g (Kyttälä & Lehto, 2008; Primi, Ferrão, & Almeida, 2010). One potential explanation is that matrix reasoning here encapsulates some of the skills necessary for the emergence of the relationship between backwards matrices and mathematics seen in Allen et al. (2020a), being a visually-based measure, and also potentially incorporating an executive component (Decker, Hill, & Dean, 2007). Executive components are argued by some to be the defining characteristic of working memory measures (see Baddeley, 1996 for an explanation), hence a more demanding executive component, greater than that in backwards matrices tasks, may be driving the relationship seen here. We also see a significant correlation between verbal working memory and vocabulary, hence a similar explanation could be applied here. Two final correlations that piqued our interest are those between both backward matrices and backward word span and vocabulary. A significant result here was unexpected, however, may reflect the strategies selected by the children to complete the tasks. If children chose verbal strategies to complete these tasks, then it would be reasonable for these tasks then to be correlated with a measure of verbal intelligence as verbal rehearsal strategies begin to appear around this age (Flavell et al., 1966; Gathercole, 1998; Henry et al., 2012).

The lack of significant predictors in any of the regression models is likely due to the small sample size used, however, raises some questions over the combination of measures used to screen children, despite the approach itself being feasible. As such, future research should seek to replicate the study with a larger sample so that firmer inferences can be drawn from the analysis. In addition, a larger sample would allow researchers to investigate potential mediators further. We aimed to scope whether the relationships identified in Allen et al. (2020a) remained following the inclusion of other well-known predictors of mathematics, with preliminary analysis suggesting the effect is negated. It is not possible to assess mediators with the current sample size to ascertain which measures are the cause of this. Finally, future research should seek to investigate the potential for non-linear relationships between the variables. The scatterplots in figure 1 suggest non-linear relationships, indicating scope for future analysis on a larger sample, particularly in relation to symbolic number sense, non-symbolic number sense, vocabulary, and backward matrices.

In conclusion, the data on the whole sample suggest the unique influence of backward matrices and backward word span on mathematics is diminished when other known cognitive predictors of mathematics are included. However, the sample size here is too small to draw firm conclusions, giving scope for future research to investigate further.

Subgroup analysis

When considering the subgroup analysis, as with the whole group analysis, the main reason for the lack of significant predictors in the regression models is likely to be the sample size. When we consider the fit index (AIC) for the models generated, the model with the best fit is that including two measures: a composite measure for verbal working memory and a composite measure for visuospatial working memory. This is harmonious with a large body of literature suggesting that working memory is a reliable predictor of educational attainment (e.g. Alloway, 2006; Alloway & Alloway, 2010), particularly in mathematics (e.g. Bull, Espy, & Wiebe, 2008).

We then went on to examine whether there was any evidence for any fundamental differences underpinning those who are a cause for concern due to their underperformance and their peers. None of the ANOVAs performed were significant, suggesting that there are no fundamental differences between the two groups in this sample. Further, the effect sizes were consistently small, apart from symbol search and backward matrices, which showed medium effects, and matrix reasoning, which showed a small-medium effect. In contrast, Andersson and Lyxell (2007) suggest that, compared to typically developing children, those with mathematics difficulties show a clear working memory deficit. They do, however, state the deficit appears to relate to the function of the central executive. It is, however, necessary to note that the children in the concern group were compared to a control group of children selected because they performed in the lower half of their class on the mathematics test, hence they are relatively low scoring themselves. These results reflect the argument made by Julian Elliott regarding dyslexia.

In "The Dyslexia Debate", Elliott amd Grigorenko (2014) argue that there are no fundamental differences between those children with a reading difficulty and those diagnosed as dyslexic. Nor are there any differences in the way these problems are remediated (Elliott &

Gibbs, 2008; Gibbs & Elliott, 2015). In short, the difference between the two groups is likely to be a matter of severity. Based on the results of this study, we would argue a similar perspective for those with mathematics difficulties. Although children in this sample were not diagnosed as dyscalculic (a specific developmental problem relating to counting and number, Kadosh & Walsh, 2007), the scores of the concern group reflect those that would be expected for dyscalculics, particularly depending on the area the child attends school. Dyscalculia is often diagnosed at different rates, and for children showing different levels of severity of mathematics difficulty, because children are diagnosed based on deviations from norms based on cognitive performance or observed behaviour, depending on the basis of the assessment (Adams, 2007; Devine, Soltész, Nobes, Goswami, & Szűcs, 2013). For example, a child from an area where most children thrive in mathematics would not necessarily have to be struggling to the same extent as a child from an area where children are generally weaker in mathematics (Peterson & Shinn, 2002). Socioeconomic status has also been shown to be an influencing factor in dyscalculia diagnoses (Gross-Tsur, Manor, & Shalev, 1996). Part of this difference must be attributed to how likely a child is to stand out as having profound difficulties depending on their peer group, of course.

In summary, based on analysis of the subgroup, no unique predictors emerge from the data for group belonging based on any of the cognitive measures taken. These results again indicate that the approach taken is feasible for predicting poor performance, however, further work is required to identify the appropriate predictors to include. Similarly, we see no significant differences in performance on the cognitive measures between the groups. As such, these preliminary findings suggest a preliminary conclusion that no fundamental differences underlie more pronounced mathematical difficulties when compared with other poor performing children. Thus, using the measures included here to highlight poor performing children may not be the most appropriate approach. However, we must be aware that these comparisons were made to children who also show relatively poor performance, hence findings may be different if using average or good performers as the comparison group. These findings lead us to question more the individual differences between children to understand the potential causes of their difficulties.

Individual analysis

Finally, we considered the individual profiles of the nine children previously identified as performing at least 1SD below age expectations in mathematics. When generating the group means and CIs necessary for individual comparison against, it was striking that the only measure that presented a mean and CI for the group that remained consistently below 0 was matrix reasoning. Taking this result with the significant correlation between matrix reasoning and mathematics from the whole group analysis suggests that the skills required to complete the matrix reasoning task may map well onto those required for mathematics tests. One possibility is the level of abstraction required for pattern completion (Bennett & Müller, 2010). Mathematics tests also represent a number of abstract concepts, thus being competent at this skill may be of benefit. A second potential explanation is as explained above, with matrix reasoning representing a predominantly visual skill, echoing the findings of a number of studies that visuospatial working memory is heavily implicated in mathematics (e.g. Mammarella, Caviola, Giofrè, & Szűcs, 2018; see Allen, Higgins, & Adams, 2019 and Passolunghi & Costa, 2019 for reviews).

Similarly, and as previously mentioned, when drawing comparisons between the concern and control groups, the individual profiles of the nine cause for concern children reflect the argument made in dyslexia research (Gibbs & Elliott, 2015). There does not appear to be a consistent deficit in the cognitive profiles of these children, particularly not one that distinguishes them from their poorly performing peers. This finding has major implications for educators by highlighting the problems with single word diagnoses. While this sample did not contain any children with a formal diagnosis of dyscalculia, it clearly demonstrates the extent of the heterogeneity inherent in diagnosing all children with sufficiently severe mathematics difficulties as dyscalculic. Further, a single word diagnosis suggests that all children with this diagnosis will be responsive to the same remediation strategies, however, it is apparent by considering the profiles illustrated in table 4 and figure 2 that this is extremely unlikely.

Finally, whilst there is a host of evidence for the influence of working memory, speed of processing, and general intelligence on mathematics, as mentioned previously, it is surprising that the child with the lowest score for the mathematics test did not show a specific deficit in any one of these areas. This is particularly interesting as the children in this group with the highest mathematics scores showed some indication of deficits. All other children did, however, show at least an indication of a potential deficit in one or more of these areas. There are, of course, other possibilities to consider regarding why such a low score may have been obtained, for example lack of investment in the study or a lack of co-operation. However, the test was completed in test conditions, in the classroom, with the child's teacher present, hence these are unlikely to be complete explanations. Further, no children appeared to be exhibiting these behaviours when tested, despite this not being formally recorded.

In conclusion, the present data suggest that this approach is feasible, provided that the measures included give a sufficiently accurate picture of the child's profile in order to make specific predictions about likely ability. The data suggests no clear and consistent deficits defining poor performance on mathematical tests, though most children do show at least some evidence of deficit in one or more of the areas measured. Further, matrix reasoning seems to have posed the greatest challenge for this group, indicating a potential influence associated with task demand. Finally, we conclude that remediation strategies for poor mathematical performance should be prescribed based on a thorough understanding of the child's cognitive profile and capabilities, rather than attempting to apply a 'one size fits all' model to the problem.

15

Study 4 Introduction: Follow Up From Study 1

This final study of the series is the two-year follow up to study 1 time 1. The question we are asking is whether it is still possible to use the working memory measures taken when the children were in Year 3 to predict their mathematics performance now that they are in Year 5. Does the model still predict unique variance in mathematics using the working memory measures? And has this relationship changed with time? There is existing evidence that working memory can be used to predict mathematics performance over time (e.g. De Smedt et al., 2009; Geary, 2011; Kyttälä, Kanerva, Munter, & Björn, 2019), with many of the findings indicating a relative shift in the contribution of the components of working memory. As such, we predicted such a shift in contributions, but were mindful to include previous mathematics performance in the model to allow us to determine how much of the variance in mathematics at time 2 can be accounted for by working memory measures that cannot be accounted for by mathematics at time 1. The following paper is published in Psychological Research (Allen, K., Giofrè, D., Higgins, S., & Adams, J. (2020). Using working memory performance to predict mathematics performance 2 years on. Psychological Research. doi:10.1007/s00426-020-01382-5; Appendix L) and was written with collaboration from Dr. David Giofrè. The paper follows the study design used in study 1, which was designed and data collected before Dr. David Giofrè joined the project.

16

Study 4: Using working memory performance to predict mathematics performance 2 years on

Abstract

A number of previous studies have used working memory components to predict mathematical performance in a variety of ways, however, there is no consideration of the contributions of the subcomponents of visuospatial working memory to this prediction. In this paper we conducted a two year follow up to the data presented in Allen et al. (2020c) to ascertain how these subcomponents of visuospatial working memory related to later mathematical performance. 159 children (M age = 115.48 months) completed the maths test for this second wave of the study. Results show a shift from spatial-simultaneous influence to spatial-sequential influence, whilst verbal involvement remained relatively stable. Results are discussed in terms of their potential for education and future research.

Introduction

Using working memory to predict mathematical attainment is an area of study that has gained a significant amount of traction in recent years. Mathematics is a broad field and there has been extensive research across a number of aspects of mathematics and working memory which has been summarised in reviews and meta-analyses, from studies of typically developing populations (Friso-van den Bos et al., 2013; Raghubar et al., 2010), to the relationship with learning difficulties in mathematics generally (David, 2012; Swanson & Jerman, 2006) and in terms of the verbal and numerical domains in particular (Peng & Fuchs, 2016). According to the multicomponent model (Baddeley & Hitch, 1974), working memory involves subcomponents relating to the processing of visuospatial and phonological stimuli. The components of working memory have been reliably linked to academic performance on a number of occasions (e.g. Alloway & Passolunghi, 2011; Holmes & Adams, 2006; Van de

Weijer-Bergsma, Kroesbergen, & Van Luit, 2015; see Peng, Namkung, Barnes, & Sun, 2016 for a review of this literature) with a reasonable amount of evidence suggesting visuospatial working memory is more influential in younger children (e.g. Caviola, Mammarella, Lucangeli, & Cornoldi, 2014; Clearman, Klinger, & Szucs, 2017; Holmes, Adams, & Hamilton, 2008). There is also a smaller, though not insignificant, amount of evidence indicating the involvement of verbal working memory (e.g. Kyttälä, Kanerva, Munter, & Björn, 2019; Wilson & Swanson, 2001); a finding we replicated at time 1 (T1) of this study (Allen et al., 2020b).

At T1, results revealed that, when compared directly to spatial-simultaneous and spatial-sequential measures, verbal numeric tasks were more predictive of mathematics in 7-8-year-old children. Similarly, Allen, Giofrè, Higgins and Adams (2020a) demonstrated that verbal working memory (non-numeric) was more predictive of mathematical performance in younger children, with a move toward visuospatial influence in older children. It is not yet fully understood, however, how these components relate specifically to mathematical attainment on a longitudinal basis. There is some evidence suggesting visuospatial working memory is influential in the prediction of mathematics over a number of years (e.g. Bull, Espy, & Wiebe, 2008; De Smedt et al., 2009; Fanari, Meloni, & Massidda, 2019; Geary, 2011; Hilbert, Bruckmaier, Binder, Krauss, & Bühner, 2019; Li & Geary, 2017), however, as indicated by Hilbert et al. (2019), it is necessary to consider the mathematics test used for the purposes of these studies. In some cases, standardised measures, in line with the curriculum of the country are used, in which case more credence can be given to the real-life applicability of the finding. However, oftentimes researchers use tests designed specifically for the purposes of their study, in which case they lack the necessary real-world application of the findings. There are also findings to the contrary indicating the importance of verbal working memory (e.g. Geary, Nicholas, Li, & Sun, 2017; Kyttälä et al., 2019), the varying influence of the subcomponents depending on the area of mathematics in question (van der Ven et al., 2013), and even that working memory is not directly predictive of mathematics (Gathercole, Brown, & Pickering, 2003), especially when other precursor measures of mathematics are included (Krajewski & Schneider, 2009b). One area that these studies do not account for is the format of the testing in each of the domains of working memory, for example, visuospatial stimuli can be shown both simultaneously and sequentially, which may have an influence on their predictive value, particularly when considering different areas and levels of mathematics.

There is growing evidence for the subdivision of visuospatial working memory into spatial-simultaneous and spatial-sequential categories, based on the presentation of the information during the encoding phase (e.g. as in Blalock & Clegg, 2010; Lanfranchi, Carretti, Spanò, & Cornoldi, 2009). Spatial-simultaneous tasks require participants to recall a visual array when all items are presented simultaneously, while spatial-sequential tasks require recall of visual locations presented sequentially, generally in a given order (e.g. Mammarella et al., 2006; Mammarella, Pazzaglia, & Cornoldi, 2008). Evidence for a double dissociation between the two subtypes of visuospatial working memory (Mammarella et al., 2006, 2018; Wansard et al., 2015) presents the possibility that deficits in these subcomponents act as a specific vulnerability for mathematical difficulties. This is particularly pertinent if there is evidence of a longitudinal predictive relationship between the subcomponents and mathematics. The relationships between the subcomponents of visuospatial working memory and mathematics are not, as yet, thoroughly understood, therefore, this paper aims to contribute to this understanding in order to develop our ability to predict mathematics performance from working memory capacity.

There are a number of issues associated with the selection of a measure of mathematics for research purposes, including, but not limited to, the applicable age range, the standardisation procedure for the test, and design purely for research purposes, all of which increase the risk of a lack of reliability and validity of the measure in a classroom setting. To bypass some of these issues, a standardised measure was chosen which was suitable for an appropriate age range, which was standardised on a UK sample, and which was designed to map directly on to the current National Curriculum for England and Wales. Mapping onto the National Curriculum means that all children involved in the study have been exposed to the same mathematical content, therefore, should have similar background experience in terms of answering the questions. The same mathematics test was used as at T1 (Access Mathematics Test). This test was selected as it covered topics appropriate for children aged 6-12, therefore, the same measure could be administered at both time points to make a direct comparison. The test has two forms, A and B, which are designed to be equal to each other in terms of both difficulty and the distribution of topics assessed (see Access Mathematics Test Handbook for this information). At T2, the alternate form was administered to that which the children had done at T1 (if form A was used at T1, form B was used at T2, and vice versa) such that children had not had previous exposure to the same questions so that their performance was not skewed in any way.

This study aims to identify whether there is a relationship between working memory measures taken in Year 3 and a mathematics measure taken in Year 5, and if so, whether the nature of this relationship is the same as when the mathematics measure was also taken in Year 3. We aim to identify which working memory predictors can predict mathematical performance in Year 5 when mathematical performance in Year 3 is taken into account. We expect to see a shift in the extent of the relative contributions of the elements of working memory, particularly between the verbal and visuospatial elements given the suggestion of a developmental shift between the two ages the children were tested at.

Method

Participants

The initial sample included 214 7-8-year-old children, however, subject attrition over the two-year period resulted in a final sample of 159 9-10-year-old children (76 male and 83 female, M age = 115.48 months, SD = 3.43). We strove to re-test as many of the original opportunity sample of children, now in Year 5, as possible. Opt-out parental consent was obtained, as with the first administration of the study, to reduce bias in the sample (Krousel-Wood et al., 2006). The study was approved by the School of Education Ethics Committee at the University of Durham. Children with special educational needs, intellectual disabilities, or neurological and genetic conditions were not included in the study. Those who did not complete the first administration phase of the study were not included in the analysis, such as children who had entered the school within the last two years.

Design & Procedure

Previously, children had been tested individually on working memory measures (spatial-simultaneous, spatial-sequential, and verbal) and mathematics as a class group in Year 3 (see Allen et al., 2020b for a full description of this phase) to form Time 1 of the study. This second phase (Time 2) of the study required only a mathematics test, therefore, children were tested as a class group. Working memory measures were not administered at this stage as the intention was to understand whether it is possible to design a measure to be administered at the beginning of formal schooling to predict whether a child is likely to encounter mathematics difficulties in the future, hence this would only be measured once. Testing was done in the child's usual classroom and with their class teacher present to minimise stress, but was completed under typical test conditions. The test was administered according to the instructions in the testing manual (see below for further explanation), with a 10-minute warning prior to the end of the test. Paper and pencil format was used and children could request a question be read aloud in order to account for those children with a lower reading ability. No further help was given as part of the reading process, nor were any numbers that may have been particularly pertinent to the question, for example "The River Nile is 3256km long. Round this to the nearest 1000km." would be read as "The River Nile is this long (point to number). Round this to the nearest this distance (point to number)". We used a correlational design to investigate the relationships between earlier working memory measures and current mathematics performance.

Measures

Working Memory

Working memory measures from T1 were used for this analysis. At T1, measures of verbal working memory (digit recall, backwards digit recall, and counting recall, as presented in the Working Memory Test Battery for Children; Gathercole & Pickering, 2001)), spatial-simultaneous working memory (4 × 3 and 4 × 4 dot matrices tasks - children were presented grids containing dots and were required to recall the positions of the dots), and spatial-sequential working memory (3 × 3 and 4 × 3 dot matrices tasks - children were presented grids in which dots appeared sequentially and were required to recall the positions of the dots in no specific order - and block recall; Corsi, 1972) were administered to all children prior to the mathematics test. See Allen, Giofrè, Higgins and Adams (2020b) for a full description of the measures taken during phase one.

Mathematics

Access Mathematics Test (AMT): The AMT is a standardised measure of National Curriculum mathematics, designed to test children aged 6 - 12 years. It, therefore, provides clear evidence for how well each child performs in individual areas of mathematics, as well as overall. The AMT covers the requirements of the National Curriculum in England and Wales, where children are required to understand number, measurement, geometry, and statistics, hence providing an ecologically valid measure of a child's school performance. Questions cover number (e.g. "the distance from New York to London is 3457km. Write this distance to the nearest 1000 kilometres"), operations (e.g. "write the missing number. __ ÷ 5 = 35"), fractions, including ratio (e.g., "peanuts cost 40p for 100g. How much does 120g of peanuts cost?"), geometry (e.g. "the point A is moved three squares to the right and two squares down. Write the coordinates of this new point A"), measures (e.g. "how many 20p coins are there in £13?"), and statistics (e.g. "this bar chart, from a spreadsheet, shows the number of pets each pupil owns. How many pupils own 2 pets or more?").

Children were read the instructions set out for the AMT, which included a time limit of 45 minutes, clarification of where to write their answer on the paper, and explanation that workings were allowed on the paper, providing their answer was clearly written in the correct space. Typical classroom test conditions were adopted throughout. Children were permitted to request questions be read aloud to them should they have difficulties so as not to disadvantage those with weaker reading abilities, however, no further explanation of the question, or what was required, was given. No discontinuation rule was employed as children were instructed to complete as many questions as they could, but that questions were also included for children much older than they were so not to worry if they could not complete them all. The total number of test items for this test is 60, with a maximum score of 60.

Data Analytic Strategy

All analyses were performed using R (R Core Team, 2018). The R program (R Core Team, 2018) with the "lavaan" library (Rosseel, 2012) was used to conduct structural equation modelling (SEM). Model fit was assessed using a variety of indices according to the criteria suggested by Hu & Bentler (1999). In particular, the chi-square (χ^2), the comparative fit index (*CFI*), the non-normed fit index (*NNFI*), the standardised root mean square residual (*SRMR*) and the root mean square error of approximation (*RMSEA*) were used to evaluate model fit, while the Akaike information criterion (AIC; the lower the better) and the chi-square difference (with results not statistically significant favouring more parsimonious models) were used to compare the fit of alternative models.

Full-information maximum likelihood (*FIML*) estimation was used to handle missing data in our analyses. This method offers unbiased estimates under missing data patterns such as missing completely at random (MCAR) or missing at random (MAR). The pattern of missingness was tested using correlations (see Kabacoff, 2015 for the rationale). Missing values at T2 were coded 1 for missing and 0 for present. This dummy variable was then correlated with our measures at T1 (i.e., working memory and mathematics). None of the correlations were particularly large or striking (*r*s < .17), which suggests that the data deviate minimally from MCAR and may be MAR. Therefore, the assumption that data are either MCAR or MAR is justified. Maximum likelihood estimation with robust (Huber-White) standard errors and a scaled test statistic was used for the analyses. This test provides robust estimates and should be preferred in every normal application using SEM (Rosseel, 2010).

The influence of age in months was taken into account by calculating standardised residuals for each variable included in this study. Residuals were calculated entering each score as the dependent variable and age as predictor (see Allen, Giofrè, Higgins, & Adams, 2020b and Giofrè & Mammarella, 2014 for a similar method).

Results

Table 1 shows correlations among variables at T1 and at T2 together with descriptive statistics for these variables.

Table 1. *Pairwise correlation matrix with raw score correlations below the leading diagonal and age covaried correlations above the diagonal, including means and standard deviations for each measure.*

	1	2	3	4	5	6	7	8	9	10
1. Simultaneous 4 x 3	—	.685*	.484*	.437*	.407*	.352*	.321*	.180*	.410*	.430*
2. Simultaneous 4 x 4	.681*	—	.416*	.433*	.407*	.305*	.289*	.122*	.397*	.408*
3. Sequential 3 x 3	.488*	.415*	—	.573*	.343*	.301*	.257*	.112	.308*	.414*
4. Sequential 4 x 3	.440*	.430*	.576*	—	.363*	.257*	.277*	.139*	.300*	.372*
5. Block recall	.416*	.406*	.349*	.368*	—	.287*	.239*	.077	.242*	.238*
6. Counting recall	.358*	.308*	.300*	.253*	.289*	—	.444*	.322*	.385*	.420*
7. Backward digit	.325*	.290*	.259*	.279*	.243*	.445*	—	.325*	.318*	.390*
8. Digit recall	.180*	.123*	.110*	.135*	.076	.325*	.325*	—	.156*	.204*
9. Math Assessment Y3	.417*	.399*	.310*	.302*	.248*	.390*	.320*	.158*	—	.832*
10. Math Assessment Y5	.420*	.407*	.411*	.369*	.232*	.413*	.387*	.202*	.823*	—
M	28.28	20.11	18.7	15.36	21.5	16.33	10.52	26.61	11.72	24.19
SD	5.99	6.85	4.72	4.23	4.09	3.99	3.08	3.51	6.64	10.140

Note. * p < .05 one tail.

The main aim of this longitudinal paper was to evaluate the impact of working memory on mathematics, controlling for the effects of mathematics at T1. To achieve this aim, SEM was used, fitting a model with three latent variables for working memory (spatial sequential, spatial simultaneous, and verbal), and two observed variables for mathematics at T1 and T2. In this model, the three correlated working memory factors were predicting mathematics at T1 and T2, while mathematics at T1 was also predicting mathematics at T2. This latter path allows us to control for potential autoregressive effects, i.e., the performance in mathematics at T2 is controlled for the performance in mathematics at T1. This model design allows us to control for the shared contribution of working memory, i.e., the effect of each working memory factor is over and above the effect of the other predictors.

The fit of the model was good, $\chi^2(27) = 20.73$, $p = .799$, $RMSEA = .000$, $SRMR = .029$, $CFI = 1.00$, $NNFI = 1.014$ (Figure 1). In this model, paths from simultaneous and verbal working memory factors to mathematics at T1 were statistically significant, while the path from sequential working memory was not. As for mathematics at T2, the path from mathematics at T1 as well as paths from sequential and verbal working memory, were statistically significant albeit with a small effect size, while the path from simultaneous working memory was not.

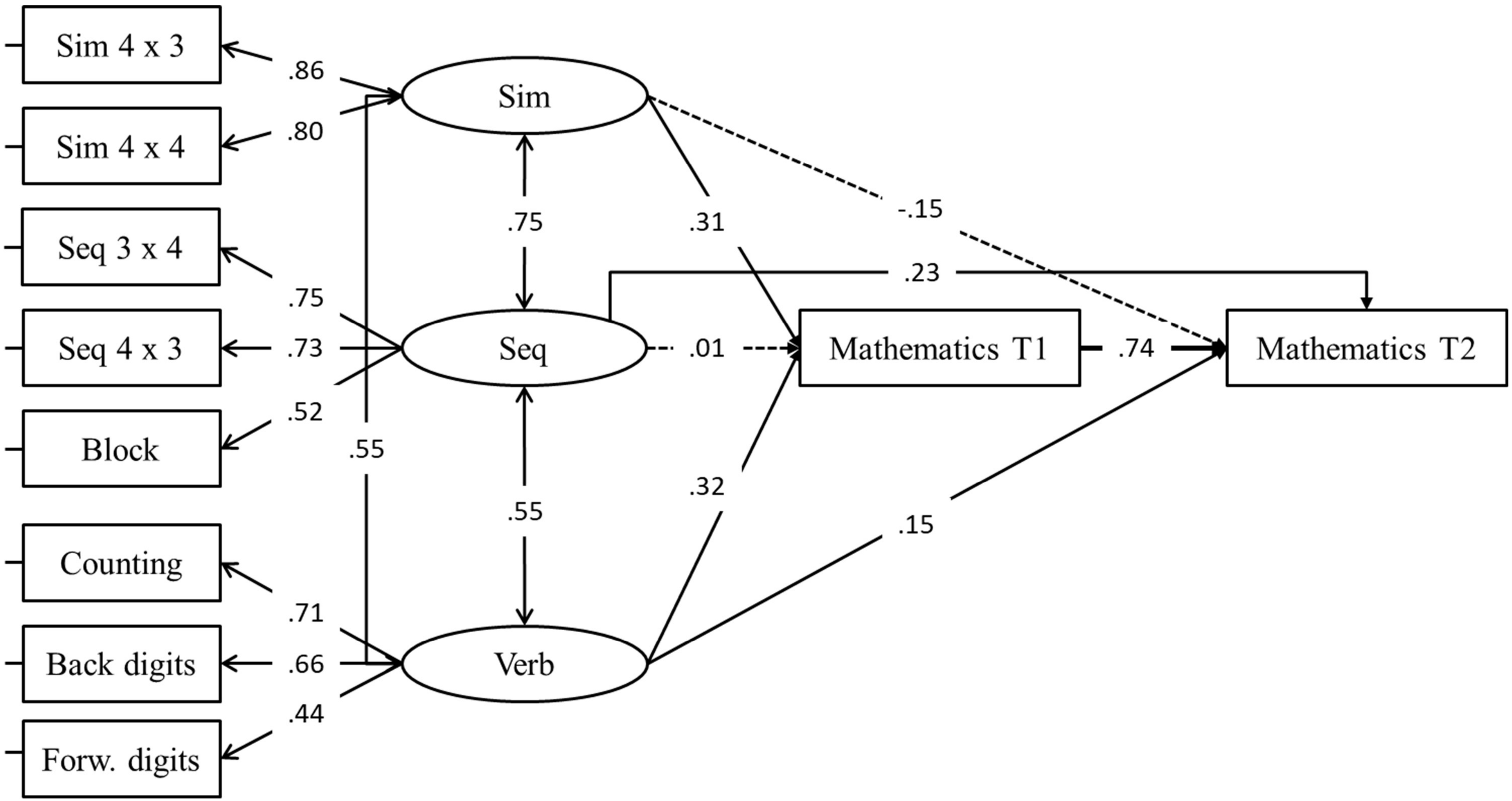


Figure 1. SEM model for working memory, mathematics T1 and T2. Solid lines represent statistically significant paths ($p < .05$).

Additional analyses

Mathematics is a broad concept, addressing for example measurement, properties, and relations of quantities (Peng et al., 2016). The test we used to evaluate mathematics includes different components, making it possible to distinguish among them. This comparison is of particular interest because it can be argued that the relation between verbal, spatial-simultaneous and spatial-sequential working memory can potentially be affected by different types of mathematics skills. It can also be argued that there might be a shift in this relationship due to a change in the curriculum (i.e., different proportions of the different domains). According to this hypothesis, one could assume that verbal-numeric working memory could have a stronger relation to word-problem-solving or to number-based mathematics skills (e.g., calculation), and visuospatial working memory to visual-related mathematics skills (e.g., geometry). To investigate this issue, we performed some additional analyses.

In a fist SEM model, similar to what we did in the aforementioned SEM model, three exogenous working memory factors (i.e. variables that are not caused by another variable in the model) were calculated (i.e., simultaneous, sequential and verbal). These working memory factors were allowed to correlate. As for the endogenous variables (i.e. variables that are caused by one or more variable in the model), rather than including the overall score for mathematics as we did before, all subdomains were included separately (i.e., number, operations, fractions including ratio, geometry, measures, statistics including probability). Residual errors of mathematics domains were also allowed to correlate, this is normal practice in SEM when tasks, as in this case, belong to the same constructs and are intrinsically related in nature, that is they share a significant portion of the variance over and above what is accounted for by working memory factors in this case. In the model, each working memory factor was independently predicting each mathematic variable. In this first model all betas were freely estimated (i.e. were supposed to be independent from each other). The fit of this model was satisfactory, $\chi^2(47) = 48.70$, $p = .405$, *RMSEA* = .013, *SRMR* = .029, *CFI* = .998, *NNFI* = 0.997, AIC = 13200.

Having established that the model provided a satisfactory fit we tested several alternative nested models in which the betas from working memory to each mathematic domain were constrained to be equal across the tasks (i.e. the relationship to working memory was considered to be similar in each individual mathematic subdomain). We took a multi-step approach, fixing one group of betas at a time. In the first model, betas from simultaneous working memory to each mathematic domain were constrained to be equal (assumed to be similar across each individual math variable). The fit of this model was similar to the previous model, $\chi^2(52) = 53.24$, $p = .426$, *RMSEA* = .011, *SRMR* = .030, *CFI* = .999, *NNFI* = 0.998, AIC = 13195. Importantly this latter model had a lower AIC, was more parsimonious (i.e. had a higher number of degrees of freedom), and was not statistically different from the previous one, $\Delta\chi^2(5) = 4.38$, $p = .4958$, meaning that this model should be preferred over the previous one. This finding indicates that increasing the complexity of the model and assuming different betas (i.e., different relationships) from the simultaneous working memory factor to each mathematic subdomain was not necessary (i.e. the simultaneous working memory factor had a similar impact on each individual mathematic task).

In a further model, we went on constraining betas from the sequential working memory factor to each mathematic subdomain to be equal. The fit of this model was similar to the previous model, $\chi^2(57) = 61.88$, $p = .306$, *RMSEA* = .020, *SRMR* = .035, *CFI* = .996, *NNFI* = 0.993, AIC = 13194. Also, in this case, this latter model had a lower AIC, was more parsimonious, and was not statistically different from the previous one, $\Delta\chi^2(5) = 7.85$, p = .1645. These findings taken overall indicate that increasing the complexity of the model and assuming different betas (i.e., different relationships) between the simultaneous and sequential factor to each mathematic subdomain was not necessary.

In a further model, we went further on constraining the betas from the verbal working memory factor to each mathematic task. The fit of this model was somewhat poorer as compared to the previous one, $\chi^2(62) = 89.36$, p = .013, *RMSEA* = .046, *SRMR* = .081, *CFI* = .975, *NNFI* = 0.964, AIC = 13211, $\Delta\chi^2(5) = 25.55$, p = .0001. Such a finding indicates the possible presence of misfit, which was examined looking at modification indices and residuals. The inspection of the model led us to free one of the betas (i.e., the link from the verbal working memory factor to the operations component). This resulted in a considerably better fit, $\chi^2(61)$ = 68.96, p = .226, *RMSEA* = .025, *SRMR* = .052, *CFI* = .993, *NNFI* = 0.989, AIC = 13192. Comparing this model with all the previous one we also established that this was the best fitting model as it had a lower AIC and was statistically superior as compared to all previous models. These findings taken together indicate that betas from simultaneous, sequential and verbal factors to each individual mathematic subdomain are similar, with only one exception (Figure 2).

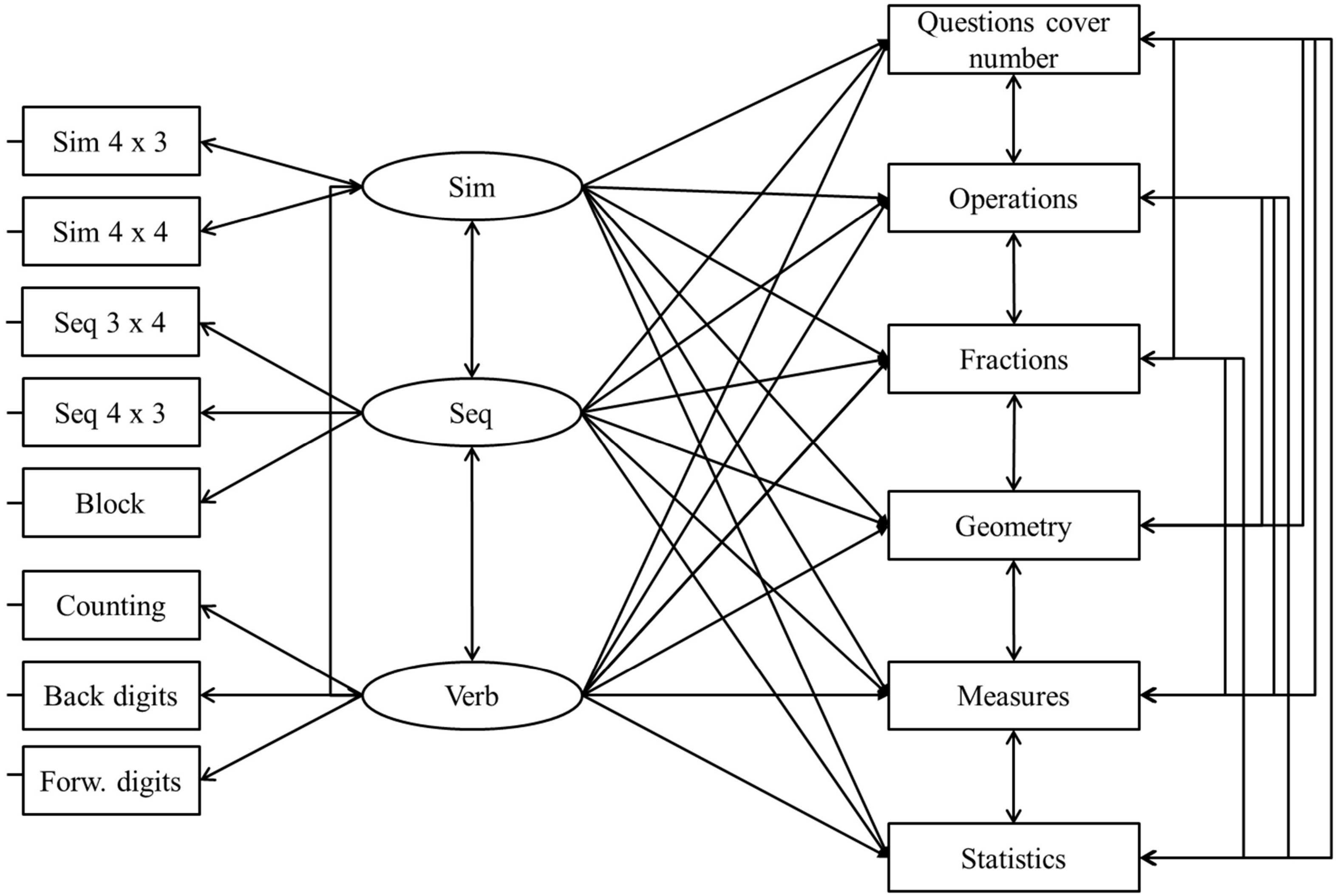


Figure 2. Theoretical model for the relationship between working memory factors with observed mathematical subtests.

Discussion

This study aimed to investigate the contributions to written mathematics made by verbal, spatial-simultaneous, and spatial-sequential working memory over the period of two years. Previous results (Allen et al., 2020c) highlighted a significant relationship between mathematics and spatial-simultaneous and verbal working memory in 7-8 year old children. Therefore, we aimed to assess whether this relationship remained stable two years later or varied as a function of age.

From the correlations table, all correlations (both normal and after covarying for age) between mathematics, measured at T1 and T2, and working memory measures were statistically significant. This suggests that working memory is related to mathematics, both at T1 and T2. With regard to our specific research question for this paper, we identify a shift in the influence of the components of working memory on mathematics. Whilst verbal working memory remains a significant predictor, spatial-simultaneous becomes non-significant and is taken over by spatial-sequential. The relationship between spatial-sequential working memory and mathematics is stronger than that between verbal working memory and mathematics, though not significantly so, however, the strongest relationship remains between mathematics at T1 and T2. It was anticipated that this would be the case, therefore, the model was built in such a way that the significant relationships identified between spatial-sequential and verbal working memory and mathematics remain after accounting for the relationship between mathematics at T1 and T2. That is, these relationships reveal the amount of variance in mathematics that we are able to account for over and above that which is predicted by previous mathematical ability. As discussed in Allen et al. (2020b), the presence of a significant relationship with verbal working memory is supported by literature suggesting verbal-numeric tasks, as are those used in this study, show a direct relation to mathematical performance (as reviewed by Raghubar, Barnes, & Hecht, 2010).

There is a possibility that the influence of spatial-sequential working memory at T2 could be due to the complexity of the task. In order to complete sequential tasks, children are required to hold the initial elements of the stimuli sequence in mind for longer before recall, which could be considered more demanding than spatial-simultaneous tasks (Rudkin, Pearson, & Logie, 2007). This requirement to hold information for longer periods of time when encoded at different time points may replicate the child's ability to handle sequentially derived information resulting from multi-step mathematics problems. Older children are more likely to encounter these types of problems in mathematics (Department for Education, 2013), therefore, spatial-sequential tasks may be more predictive of older children's mathematical ability (Allen et al., 2020b), particularly if the proportion of multi-step versus single-step problems encountered also increases with age, as is often the case. As a result, it may be that spatial-sequential working memory is more strongly related to mathematics than verbal working memory due to the way the information is encoded. Similarly, Caviola, Colling, Mammarella and Szűcs (2020) suggest that spatial working memory may provide the mental workspace required to complete mathematics tasks, which is likely to be increasingly important in multi-step tasks.

Understanding the task demands of the working memory tasks themselves, particularly the spatial-sequential tasks, cannot be an influencing factor in their relationship with mathematics in this study because working memory measures were taken at T1 only, as opposed to being repeated at T2. Only the mathematics measure was repeated at T2. This calls into question the evidence that high and low ability children in mathematics are not distinguishable by their spatial-sequential working memory (Bull, Johnston, & Roy, 1999), as the current result would suggest this may be possible. There is, however, an alternative argument by Andersson and Lyxell (2007), D'Amico and Guarnera (2005), McLean and Hitch (1999) that our current results support. Caution should be applied when trying to define distinct groups of children in mathematics based on their cognitive profile, as Allen, Higgins and Adams (2020) suggest there is little evidence of a distinct profile of poor performers in mathematics in those without a diagnosis of mathematics difficulties.

Based on previous research suggesting a declarative shift (see Schneider, 2008 for a review of this literature), it is surprising that spatial-sequential working memory remains so influential in 9-10 year old children. It has long been considered that younger children rely on using visuospatial working memory for mathematics (Van de Weijer-Bergsma et al., 2015), potentially because it acts as a mental 'checker' or allows them to use visual strategies that young children rely on so heavily. When children are first introduced to mathematical concepts, wherever possible the concept is made concrete through the use of tangible examples with blocks or counters, for example. This is done to give the children a concrete, visible reference point for the concept that they are able to interact with (e.g. draw on, rotate). Once they understand the material well enough, the scaffolding of concrete examples is slowly removed to make the work more abstract, using less tangible representation. By following this pattern, it is clear to see why the suggestion is made that children will rely more on visuospatial working memory in their younger years, before making the transition to using verbal working memory resources when they are older. However, the group of children used in this study are older than the age at which this declarative shift is predicted to take place (around 7 years of age, Schneider, 2008), therefore suggesting a shift of this nature may not tell the whole story.

One potential explanation relates to the relative lack of evidence regarding the individual contributions of the subtypes of visuospatial working memory to mathematical performance. Although not a definitive claim, a meta-analysis by Allen, Higgins and Adams (2019) suggests some influence of the type of visuospatial working memory measured on the magnitude of the effect size measured in studies relating to mathematics. This synthesis identified that the relationship between spatial-sequential working memory and mathematical reasoning (problem solving; a large portion of the mathematics test used in this study) had not previously been investigated. As such, this paper may go some way to shedding light on this relationship, highlighting a lack of a thorough understanding of the interplay between mathematics and the subtypes of visuospatial working memory previously. This is notable because the involvement of elements of visuospatial working memory in older children is supportive of other recent findings (Allen et al., 2020b). Unlike previous work suggesting a fundamental shift in the reliance on components of working memory for mathematics, the results of this study, taken as a whole, suggest verbal working memory makes a relatively stable contribution to performance, with the variability emerging from the involvement of the components of visuospatial working memory, shifting from simultaneous to sequential influence (see Allen et al., 2020b for further information on T1 of this study).

It is unlikely, though not impossible, that the shift we see in the involvement of working memory is due to the cognitive load imposed by the task as tasks are always visible and children have the opportunity to write down any workings or intermediate results, and so are not required to hold these items in mind. However, there is the possibility that children, particularly those who are anxious for example, may face more difficulties under timed conditions (Ashcraft & Moore, 2009; Carey, Hill, Devine, & Szücs, 2016; Onwuegbuzie & Seaman, 1995). There is also some evidence that children who have poor working memory are also poor at comprehending text (e.g. Carretti, Cornoldi, De Beni, & Palladino, 2004). Similarly, task instructions are always present meaning children have the opportunity to break tasks down into smaller chunks, though those with particularly poor working memory may have difficulties with keeping their place in the instructions (Alloway, 2006; Gathercole & Alloway, 2004). Due to the nature of the paper layout, extraneous cognitive load is relatively low as information is presented alongside the associated question and graphs and diagrams are interspersed through the text in the most appropriate place. There may be some influence of cognitive load due to the increased number of multi-step questions designed for older children requiring the maintenance of intermediate steps (Sweller, 1994), but this should be minimal in this case and is unlikely to fully explain the results found.

The proportions of questions concerning the different domains of mathematics could potentially influence the results over time, even though children completed the same longitudinal test (albeit the opposite paper at T2, balanced exactly for difficulty and weightings towards the different domains). All of the questions were included on the paper at T1, and some children made attempts at these, however, children will have been able to access a greater number of these questions at T2 following two years of extra schooling. There is no evidence from a visual search of the frequency of questions relating to each question type that this changes over the course of the test. If this were the case, it may be that working memory influence shifts as a direct consequence of more questions being asked that tap different working memory components later in the paper, thus only older children will be able to access them. This is not the case. As such, it follows that, when developing a screening measure, children should be screened on measures that are predictive over longer periods of time. It is important to include shorter term predictors of mathematics as well to pick up children who are likely to fall behind immediately, however, the focus should be on longer term predictors.

As with T1 of this study, there are some inherent limitations. Primarily, the use of a verbal-numeric measure of verbal working memory. Verbal numeric working memory has been shown to demonstrate a different relationship to mathematics than verbal working memory measures using stimuli not relating to numbers (see Raghubar et al., 2010 for a review of this literature). After highlighting this as an issue at T1, Allen et al. (2020a) found a similar pattern of results using non-numeric verbal stimuli. The inclusion of only typically developing children has not, however, been addressed at this time as a clear understanding of typical development is necessary before investigating the nature of the relationship in atypical samples, such as those with diagnosed mathematics difficulties. As a result, we therefore remain unable to compare the development of typical and atypical populations to assess any differences.

In this paper we have also attempted to distinguish between different mathematics domains at Y5. Intriguingly, the relationship between simultaneous and sequential working memory factors with the different mathematic subdomains seems to be quite similar. A recent meta-analysis by Peng et al. (2016) tested the relationship between working memory and different mathematics domains, demonstrating some small variations in terms of the correlations between mathematic subdomains (from .23 to .37). In fact, one could expect, for example, geometry to draw more on visuospatial skills. However, geometry seems to be a very complex domain involving several complex abilities (Mammarella, Giofrè, & Caviola, 2017). One possibility is that our results at Y5 are influenced by the nature of the geometry tasks at this stage in the curriculum. In a similar study, Giofrè, Mammarella and Cornoldi (2014), with a sample of 4th and 5th graders, found that working memory, independent of the modality, had the highest correlation with geometry. This finding was explained by the authors arguing that formal education in geometry, at this stage, involves both visuospatial and verbal materials (such as texts, definitions, formulae, and theorem). Therefore, the absence of the stronger influence of visuospatial working memory is not necessarily surprising. As for the verbal working memory factor, the pattern was rather similar but with one exception.

Results reported in the present paper show that verbal working memory has explanatory power in all mathematics domains. Intriguingly, the link between verbal working memory and a specific component (i.e. operations) seemed to be higher as compared with the other tasks. It could be argued that the manipulation of operations could draw on verbal and visuospatial working memory to a large extent (Caviola et al., 2012; Van de Weijer-Bergsma et al., 2015).

Future research should seek to continue to address the limitations presented here, as well as to build upon the findings presented to continue to develop our understanding of the relationships between the components and subcomponents of working memory and mathematics. Once this understanding has been developed, researchers can begin to work with atypical populations to try to ascertain whether these populations differ from typical populations in the ways in which working memory contributes to task completion. There are clear implications for education providers and researchers as, in developing our understanding of this area, we will be able to use this knowledge to support children who have difficulties in mathematics through supporting their working memory. By understanding which elements of working memory are most important for mathematics at different ages, educators will be able to provide targeted support for children in the form of aids and alternative methods where necessary.

In conclusion, this study confirmed that it is possible to predict mathematics using working memory data gathered two years previously, however, that the specific nature of the relationship changes over time. Spatial-sequential and verbal working memory tasks are predictive of 9-10-year old's' performance in mathematics, as opposed to spatial-simultaneous and verbal measures in the same children at 7-8 years of age.

17

Discussion of the Findings of the Studies

This project comprised a number of individual studies in order to achieve the main research aim of furthering our understanding of the relationship between working memory and mathematics in primary school aged children. The results showed some consistency with previous literature, but simultaneously indicated some discrepancies that warrant further investigation.

Review of this project – first steps

Initially, a systematic review (see chapter 6) was carried out to better understand the existing literature. This review highlighted the importance of sample size in determining an accurate estimate of effect size. It also demonstrated that standardised mathematics measures are associated with larger effect sizes. This is atypical as researcher designed tests tend to lead to larger effect sizes (Cheung & Slavin, 2016) and is likely to be due to the thorough testing applied to each specific area of mathematics. It is also likely associated with the rigorous procedures employed when standardising a measure. This is, however, speculation as the paper did not cover a full content analysis of standardised mathematics tests. Overall, this is encouraging evidence for the use of standardised measures in both research and academic assessment. Further, the systematic review suggested that the type of visuospatial working memory (simultaneous or sequential; e.g. Maennamaa, Kikasb, Peets, & Palu, 2012; Mix et al., 2016 for simultaneous or Soltanlou, Pixner, & Nuerk, 2015; Wiklund-Hörnqvist, Jonsson, Korhonen, Eklöf, & Nyroos, 2016 for sequential) or maths (numerical operations or mathematical reasoning; e.g. Bresgi, Alexander, & Seabi, 2017; Martin, Cirino, Sharp, & Barnes, 2014 for numerical operations or Campos, Almeida, Ferreira, Martinez, & Ramalho, 2013; Passolunghi & Mammarella, 2010 for mathematical reasoning) do not influence the magnitude of the effect size, thus suggesting that their influence is stable across these areas.

However, this highlights the need to understand the relationships between subdomains of mathematics over time to ascertain whether they are stable or fluctuate, as would be expected as children learn and change strategies (e.g. Van de Weijer-Bergsma, Kroesbergen, & Van Luit, 2015; van der Ven, van der Maas, Straatemeier, & Jansen, 2013).

In order to do this, study one (see chapter 8) was established as time one of a longitudinal study investigating the relationships between the components of working memory and mathematics. In order to understand which elements of working memory are important for inclusion in a screening measure designed to be administered early in a child's school career to predict their later mathematics ability, we needed to establish a clear picture of the changes in the predictive ability of each component over time in the same children. By understanding these changes, we will then be able to target the screening measure at the necessary cognitive abilities for a more accurate long-term prediction of mathematical ability. Results of this time point, with children aged seven and eight years old (Year 3 of Primary School), showed that verbal measures contribute the largest portion of unique variance of mathematics, with spatial-simultaneous measures showing some predictive contribution, but not spatial-sequential. We also assessed how each of the components of working memory related to each of the components of mathematics (see chapter 10). This analysis confirmed the pattern we had anticipated seeing in the data, with verbal working memory having a greater influence on mathematics topics typically dependent on words and numbers, for example understanding and applying mathematics, and counting and number. On the other hand, for those more typically visual components, visuospatial working memory accounted for a larger portion of the variance. The tasks used in this study may not have been ideal as the young children did struggle to understand some of the instructions, however, we saw no floor effects so this explanation is unlikely to fully explain the findings. This study led us to suggest future research should follow a similar structure using non-numeric stimuli as the numeric stimuli used here may have influenced the relationships, particularly since the academic discipline in question was maths, which is predominantly based on numbers in primary school.

Verbal-Numeric Working Memory

The debate of the influence of verbal versus verbal-numeric working memory on mathematics is one that shows no coherence as yet and so requires further investigation (see Raghubar, Barnes, & Hecht, 2010 for a review of this literature). The disagreement arises because many verbal working memory tasks used to assess capacity involve numbers. Whilst this is commonplace, it is also potentially tapping a domain specific ability, therefore, a closer relationship with other number-based tasks (here mathematics) would be expected. There are a small number of studies available on the predictive nature of verbal-numeric working memory, lending support to verbal-numeric working memory being more predictive of mathematics, as well as showing no difference from the predictive ability of verbal working memory. Peng and Fuchs (2014) stated that verbal-numeric working memory deficits were shown in all learning difficulty groups in their sample, however, that those with specific mathematics difficulties showed more severe verbal-numeric working memory deficits. They argue that this is indicative of a distinct underlying component of working memory that predisposes an individual to mathematics difficulties. It also suggests that a specific verbal-numeric deficit in those with mathematics difficulties highlights the domain-specific nature of working memory. Though this is also contested, (see Chapter 4, p.68 for an explanation). Interestingly, learning difficulty severity did not mediate the relationship, nor did the type of academic screening, indicating that the underlying deficit may not be entirely attributable to a verbal-numeric deficit. With regard to prediction studies, better digit span has been linked to children having a "head start" in mathematics that is maintained over the first three years of school (Bull et al., 2008). However, Rasmussen and Bisanz (2005) suggest that the relationship is not stable and shifts from visuospatial to verbal-numeric by grade one. In which case, this so called “head start” would not be visible on the child’s entry to school. It also lends support to the argument that we see a developmental shift in working memory involvement from visuospatial to verbal, however, that this happens much earlier than first thought and is more influenced by numeric information. As such, children may be relying on verbal-numeric working memory from much earlier than previous findings suggest verbal working memory becomes important.

Similarly, Dark and Benbow (1994) identified a positive relationship between mathematics SATs tasks and working memory tasks involving digits, however, verbal SAT measures only correlated with word-based working memory tasks. They identified a strong relationship between verbal-numeric tasks and mathematical precocity, suggesting that there are “underlying differences between verbally and mathematically precocious youth in how different types of stimuli are represented in memory”. This study is influential in suggesting that mathematical precocity could also be predicted by advantages in verbal-numeric working memory as well as mathematical difficulties. In older children though, visual and verbal working memory are predictive in seven year-olds when verbal working memory is measured using a combination of numeric and non-numeric measures (Alloway & Passolunghi, 2011). However, this relationship is not maintained in eight-year-olds where verbal working memory, including some verbal-numeric measures, is not predictive of mathematical performance. Even with non-numeric verbal working memory tasks, mathematical computation can be predicted by verbal working memory, according to Wilson and Swanson (2001), who demonstrated that this relationship was not age-dependent. Findings of this nature, suggest that verbal-numeric working memory is no more predictive of mathematics performance than non-numeric verbal working memory and that the differences stem from underlying differences in the tasks administered, rather than the component of working memory they are accessing. This suggestion is supported by other findings, such as those of Simmons, Willis, and Adams (2012) who demonstrated the predictive ability of non-numeric verbal working memory for predicting Year 3 multiplication. Their findings suggest the relationship may be mediated by the component of mathematics being investigated and that further exploration of the relationship between verbal-numeric working memory and the components of mathematics could help to establish a sense of coherence in this debate. Further work should also seek to investigate the longitudinal stability of the relationship, which could be contributing to the lack of consensus in the literature.

Review of this project – moving on

Following the conclusions from the first study that future research should seek to understand the relationships with non-numeric working memory, we designed the second study to do just that, this time with children aged six to 10 years. As previously mentioned, there is considerable debate around the potential differences in the contributions of verbal (e.g. Simmons et al., 2012; Wilson & Swanson, 2001) and verbal-numeric (e.g. Peng & Fuchs, 2014; Rasmussen & Bisanz, 2005) working memory to mathematics. Therefore, we included only word-based verbal working memory measures in this study (see chapter 12) to understand how the change in stimuli affected the variance accounted for by the components of working memory in primary school children. We also sought to understand whether this specific relationship changed over the course of primary school. The results echoed those from study 1, paper 1 (see chapter 8), showing that both verbal and visuospatial working memory make unique contributions to mathematics performance. We suggested here that the influence of the numeric component might not be as great in comparison to non-numeric verbal tasks as some literature suggests (e.g. Raghubar et al., 2010), meaning that the use of either kind of stimulus is not likely to be detrimental. Verbal working memory accounted for a larger proportion of the variance over the whole sample, however, the magnitude of the correlation changes with age. Complex visuospatial working memory tasks become more strongly correlated with mathematics in older children. The magnitude of the verbal working memory correlations remains relatively stable with age. These results are indicative of some potential longitudinal changes in the influence of working memory, in line with suggestions made by De Smedt et al. (2009) that we see a developmental shift in the contributions of working memory to mathematics. However, we demonstrated the opposite shift to that predicted by Bull et al. (2008) and Holmes and Adams (2006) as the shift appeared to be from verbal towards visuospatial involvement. Holmes and Adams (2006) suggested that this may be due older children reverting to visuospatial strategies for more complex tasks, stemming from the idea of using a mental model to support their performance. It is likely that the results were influenced somewhat by the difficulty of the tasks as the younger children struggled to understand the instructions for the dual tasks, however, this is unlikely to be the sole explanation for the findings. The study does, however, highlight the need to follow the same children over time to identify the individual differences in the involvement of working memory.

From this sample of children, a subsample of poorly performing children in mathematics was selected to establish whether it was possible to identify a subset of extremely week children from their cognitive profile (see chapter 14). This does appear to be a feasible method, though the study highlights the need to assess the full cognitive profile to fully understand their deficits. We saw no consistent deficits in the profiles of the children highlighted as a cause for concern that set them apart from those who performed poorly, but were not in this group. There was, however, a general pattern of deficit on more than one cognitive subtest across all nine children who presented as a cause for concern. From these results we suggest that remediation strategies should be administered on the full understanding of the child's cognitive profile in order to suit their strengths and weaknesses.

The final study (see chapter 16) formed time two of the initial longitudinal study to assess whether predicting mathematics performance from working memory measures completed two years earlier was possible. The results show that the specific relationships between working memory components and mathematics change over time; spatial-simultaneous is no longer a significant predictor of mathematics, however, spatial-sequential is. We argue that one possible explanation for this finding is the way information is encoded during mathematics tasks with visuospatial working memory providing the required mental workspace (Caviola et al., 2020). As was the case for time one, verbal working memory remained a significant predictor, though the strength of its relationship with mathematics weakened slightly.

Differentiating visual from spatial working memory

The terms visual, spatial, and visuospatial have been used interchangeably throughout this thesis, however, there is an unresolved debate in the literature that visual and spatial working memory can be separated. Researchers who argue for a differentiation between visual and spatial working memory generally define visual working memory as that which concerns the sensory, visual appearance of an object, versus the location or the "environmental co-ordinates" used to define spatial working memory (Ventre-Dominey et al., 2005; Zimmer, 2008). It follows that spatial relations differ from visual properties, however, it is difficult to extend the differentiation through to execution as executing one without the other is not possible. Whenever visual stimuli are presented, the participant will encode both the visual and spatial properties of the stimuli, even if that is only to note that all stimuli are randomly filled black squares. Darling, Della Sala and Logie (2007) argue that there is evidence for an experimental double dissociation between visual and spatial working memory, using memory for appearance and location, using a dual task paradigm. This suggests that it is experimentally possible to access visual and spatial working memory in isolation, however, it is unclear whether this finding is generalisable to the real world when stimuli are not presented in such a controlled manner.

A similar finding was presented for interference, demonstrating that spatial tasks were more disrupted by spatial interference and visual tasks were more interrupted by visual interference (Klauer & Zhao, 2004). This study also presented a double dissociation, however, interpreting these results as evidence for a double dissociation between visual and spatial working memory depends on being able to reliably and clearly differentiate between the types of tasks used as visual and spatial. In an attempt to do this, Tresch, Sinnamon and Seamon (1993) used a movement discrimination task as spatial interference and colour discrimination as visual interference. They found similar evidence to Darling et al. (2007) and Klauer and Zhao (2004) regarding a double dissociation whereby spatial tasks suffered more interference as a result of movement discrimination tasks and visual tasks suffered more interference from colour discrimination tasks. This finding agrees with other research that the two types of working memory may be performed by different systems, but provides no concrete evidence that it is possible to display one type of information without the other as both rely on the same perceptual system. That is, the visual field and the visual representation are distinct, however, they are processed through the same perceptual system.

Neuroscientific evidence supports the idea that visual and spatial working memory are processed through different systems within the brain. Previous to their paper on a behavioural double dissociation, Darling, Della Sala, Logie and Cantagallo (2006) found evidence for this idea of retention in different subsystems for visual and spatial information, as did Vergauwe, Barrouillet and Camos (2009). Rather than presenting information in a different manner, they employed different tasks for measuring visual and spatial working memory, using symmetry judgement as a spatial task and colour discrimination for visual. Contrary to the findings of Tresch et al. (1993), Vergauwe et al. (2009) demonstrated that visual processing interferes with spatial maintenance and vice versa, thus suggesting that it is difficult to differentiate between the two. This is even more likely outside of the laboratory where conditions are not controlled as tightly. Their findings occurred when more information was required to be processed in the same time frame, meaning cognitive load was high, and lends support to the domain general view of working memory. Taken together, this pattern of interference suggests that there may be different neurological pathways for visual and spatial working memory, but resources for processing and storage are shared.

In further support of this Rudkin, Pearson and Logie (2007) demonstrated that sequential and simultaneous visuospatial tasks suffer interference from verbal tasks, indicating the likelihood that their response is co-ordinated by a common mechanism. It is possible, given the findings of Vergauwe et al. (2009) that the same is true within the visuospatial domain. There are studies available that specify the brain regions identified as responsible for the visual and spatial elements of visuospatial working memory, though these studies don't always cohere entirely. For example, Zimmer (2008) suggests spatial working memory is controlled by the parietal cortex, whereas, Ventre-Dominey et al. (2005) states it is the dorsal parieto-occipital and prefrontal cortices. Similarly, there are discrepancies for visual working memory, with Zimmer (2008) suggesting the ventral occipital cortex, while Ventre-Dominey et al. (2005) quotes the ventral stream of the temporo-occipital cortex, prefrontal cortex, and the striatum. Zimmer (2008) therefore suggests that the neurological underpinnings of visual and spatial working memory lie in the activation of either the ventral occipital or parietal cortex, respectively, versus the distinction made by Ventre-Dominey et al. (2005) regarding ventral or dorsal activation. Interestingly, the papers agree on the involvement of the prefrontal cortex which Zimmer (2008) explicitly states controls both visual and spatial working memory. The consistency of these findings lends support to the argument that it is difficult to differentiate between the two when the information is brought together to be processed.

We may also consider whether these types of working memory can be distinguished through assessment and, therefore, whether it is a useful distinction to make. Vicari, Bellucci and Carlesimo (2003) considered how well adults with Williams Syndrome performed on tasks of visual and spatial working memory compared to typically developing controls. They used a computerised adaptation of the Corsi block task (1972) for accessing spatial memory, and a colour discrimination task for visual memory. Interestingly, Williams Syndrome adults performed significantly worse on tasks of spatial span compared to the typically developing, age-matched controls. However, their visual span was equal to controls. These results suggest that it may be possible to use spatial span to identify specific learning difficulties if a similar pattern emerges for these kinds of difficulties. To this end, Passolunghi and Mammarella (2010) found that poor problem solvers failed spatial tasks, but did not show deficits on visual or phonological tasks. They argue that this finding is due to the participants' ability to manipulate information, rather than their recall of visual details, however, matrix tasks were used for both conditions. Using the same tasks for both conditions, requiring participants to recall to location of a specific object in the matrix for visual tasks, suggests this explanation might not be completely accurate and that visual and spatial working memory may be much more difficult to distinguish.

On balance of the evidence presented above, it seems likely that visual and spatial working memory can be distinguished based on neuroscientific evidence and some behavioural evidence, however, further research is necessary to determine the accuracy and usefulness of this distinction with regard to educational performance.

The findings presented in this series of studies demonstrate the consistent positive influence of working memory on mathematics, showing how this contribution remains both stable across age (when considering verbal working memory) and fluctuates (when considering simultaneous and sequential visuospatial working memory). Study 2 particularly highlights the need for further research to understand the relationship over development in the same sample longitudinally to ascertain how best to use early working memory as a predictive measure for future mathematical attainment.

Future Research

One logical direction to move forward with research using cognitive measures to predict academic attainment is to develop an early screening measure. There are a small number of studies available that have already started to investigate this avenue, however, none have done so entirely successfully to date. The current screeners available rely on number-based measures as predictors of mathematics, for example number sense or basic number processing (e.g. Chard et al., 2005; Geary, Bailey, & Hoard, 2009; Gliga & Gliga, 2012; Jordan, Glutting, & Ramineni, 2008; Jordan, Glutting, Ramineni, & Watkins, 2010; Olkun, Altun, Gocer Sahin, & Kaya, 2016; Seethaler & Fuchs, 2010). Some of these studies even claim that these measures are better than alternatives (Geary et al., 2009; Gliga & Gliga, 2012), or that direct measures are better at predicting mathematics performance and differentiating students (Chard et al., 2005; Kelly & Peverly, 1992). Aside from the measures used, another problem presented by some of the screening studies available to date is the age of the children they screen. Number-based tasks require a degree of numeracy to have developed in the child, potentially confounding their use in a screening measure for mathematics. A number of the studies screen children who have been in school for a number of years (Fuchs et al., 2011; Gliga & Gliga, 2012), meaning that it is likely too late to prevent the child developing mathematics difficulties. The problem is likely to have already developed by the time children have been in school for a number of years, so it is better to screen early and put teachers in a position of being able to prevent problems from developing. Further, Olkun et al. (2016) also demonstrate that screening on number ability does not predict first grade mathematics performance, so is not predictive until the children are older. Whilst this is useful for long-term predictions, it suggests that these kinds of screening measures are unable to access the underlying abilities children's mathematical development relies on sufficiently well enough to effectively screen.

The main drawback of using number-based measures for screening for mathematics difficulties that is likely to cause a problem for effective screening is the lack of resolution of the debate around verbal-numeric and verbal working memory, as discussed above. Since there is no clear understanding of how verbal and verbal-numeric working memory may relate to mathematics differently, it is preferable to use measures that are unrelated to mathematics. Measures such as those used in the feasibility study of this project present one potential solution to this problem by using entirely word-based measures of verbal working memory in order to avoid any exaggerated predictions arising from the use of verbal-numeric measures until their influence is better understood. By using non-numeric, non-mathematical measures, and instead accessing the child's cognitive profile, these measures are able to be used to identify where interventions may be most effective. Such a profile will also help to isolate the underlying causes of the child's mathematical difficulties. Developing our understanding of the underlying cognitive correlates of poor performers in mathematics will also further our understanding of the potential causes of mathematics difficulties, or dyscalculia as it is sometimes diagnosed.

18

References

Abbott, D. (2015). Digital curation and doctoral research. *International Journal of Digital Curation*, *10*(1), 1–17.

Adams, A.-M. (1996). Phonological working memory and spoken language development in young children. *The Quarterly Journal of Experimental Psychology Section A*, *49*(1), 216–233.

Adams, E. J., Nguyen, A. T., & Cowan, N. (2018). Theories of working memory: Differences in definition, degree of modularity, role of attention, and purpose. *Language, Speech, and Hearing Services in Schools*, *49*(3), 340–355. Retrieved from https://europepmc.org/articles/pmc6105130

Adams, J. W. (2007). Individual differences in mathematical ability: genetic, cognitive and behavioural factors. *Journal of Research in Special Educational Needs*, *7*(2), 97–103.

Allen, K., Giofrè, D., Higgins, S., & Adams, J. (2019). Working memory predictors of written mathematics in 7- to 8-year-old children. *Quarterly Journal of Experimental Psychology*, 1747021819871243. doi:10.1177/1747021819871243

Allen, K., Giofrè, D., Higgins, S., & Adams, J. (2020a). Using working memory performance to predict mathematics performance 2 years on. *Psychological Research*. doi:10.1007/s00426-020-01382-5

Allen, K., Giofrè, D., Higgins, S., & Adams, J. (2020b). Working memory predictors of mathematics across the middle primary school years. *British Journal of Educational Psychology*, *n/a*(n/a). doi:10.1111/bjep.12339

Allen, K., Giofrè, D., Higgins, S., & Adams, J. (2020c). Working memory predictors of written mathematics in 7-to 8-year-old children. *Quarterly Journal of Experimental Psychology*, *73*(2), 239–248.

Allen, K., Higgins, S., & Adams, J. (2019). The Relationship between Visuospatial Working Memory and Mathematical Performance in School-Aged Children: a Systematic Review. *Educational Psychology Review*. doi:10.1007/s10648-019-09470-8

Allen, R. J., Baddeley, A. D., & Hitch, G. J. (2006). Is the binding of visual features in working memory resource-demanding? *Journal of Experimental Psychology: General*, *135*(2), 298.

Alloway, T. (2012). *Can interactive working memory training improve learning? Journal of Interactive Learning Research* (Vol. 23).

Alloway, T. P. (2006). How does working memory work in the classroom? *Educational Research and Reviews*, *1*(4), 134–139. Retrieved from https://academicjournals.org/journal/ERR/article-abstract/475D3083412#.XSxScAJSHIY.mendeley

Alloway, T. P., & Alloway, R. G. (2010). Investigating the predictive roles of working memory and IQ in academic attainment. *Journal of Experimental Child Psychology*, *106*(1), 20–29. doi:https://doi.org/10.1016/j.jecp.2009.11.003

Alloway, T. P., Gathercole, S. E., Kirkwood, H., & Elliott, J. (2009). The Cognitive and

Behavioral Characteristics of Children with Low Working Memory. *Child Development*, *80*(2), 606–621. Retrieved from http://www.jstor.org/stable/29738637

Alloway, T. P., Gathercole, S. E., & Pickering, S. J. (2006). Verbal and Visuospatial Short-Term and Working Memory in Children: Are They Separable? *Child Development*, *77*(6), 1698–1716. doi:10.1111/j.1467-8624.2006.00968.x

Alloway, T. P., Kerr, I., & Langheinrich, T. (2010). The effect of articulatory suppression and manual tapping on serial recall. *European Journal of Cognitive Psychology*, *22*(2), 297–305. doi:10.1080/09541440902793731

Alloway, T. P., & Passolunghi, M. C. (2011). The relationship between working memory, IQ and mathematical skills in children. *LEARNING AND INDIVIDUAL DIFFERENCES*, *21*(1), 133–137. doi:10.1016/j.lindif.2010.09.013

Aluja-Fabregat, A., Colom, R., Abad, F., & Juan-Espinosa, M. (2000). Sex differences in general intelligence defined as g among young adolescents. *Personality and Individual Differences*, *28*(4), 813–820. doi:https://doi.org/10.1016/S0191-8869(99)00142-7

Andersson, U. (2008). Working memory as a predictor of written arithmetical skills in children: The importance of central executive functions. *British Journal of Educational Psychology*, *78*(2), 181–203. doi:10.1348/000709907X209854

Andersson, U., & Lyxell, B. (2007). Working memory deficit in children with mathematical difficulties: A general or specific deficit? *JOURNAL OF EXPERIMENTAL CHILD PSYCHOLOGY*, *96*(3), 197–228. doi:10.1016/j.jecp.2006.10.001

Anobile, G., Arrighi, R., Castaldi, E., Grassi, E., Pedonese, L., Moscoso, P. A. M., & Burr, D. C. (2018). Spatial but not temporal numerosity thresholds correlate with formal math skills in children. *Developmental Psychology*, *54*(3), 458.

Ansari, D., & Karmiloff-Smith, A. (2002). Atypical trajectories of number development: A neuroconstructivist perspective. *Trends in Cognitive Sciences*, *6*(12), 511–516.

Antell, S. E., & Keating, D. P. (1983). Perception of numerical invariance in neonates. *Child Development*, 695–701.

Arsalidou, M., & Taylor, M. J. (2011). Is 2+ 2= 4? Meta-analyses of brain areas needed for numbers and calculations. *Neuroimage*, *54*(3), 2382–2393.

Ashcraft, M. H., & Krause, J. A. (2007). Working memory, math performance, and math anxiety. *Psychonomic Bulletin & Review*, *14*(2), 243–248. doi:10.3758/BF03194059

Ashcraft, M. H., & Moore, A. M. (2009). Mathematics Anxiety and the Affective Drop in Performance. *Journal of Psychoeducational Assessment*, *27*(3), 197–205. doi:10.1177/0734282908330580

Ashkenazi, S., Rosenberg-Lee, M., Metcalfe, A. W. S., Swigart, A. G., & Menon, V. (2013). Visuo–spatial working memory is an important source of domain-general vulnerability in the development of arithmetic cognition. *Neuropsychologia*, *51*(11), 2305–2317. doi:https://doi.org/10.1016/j.neuropsychologia.2013.06.031

Atkinson, R. C., & Shiffrin, R. M. (1968). Human Memory: A Proposed System and its Control Processes11This research was supported by the National Aeronautics and Space Administration, Grant No. NGR-05-020-036. The authors are indebted to W. K. Estes and G. H. Bower who provided many valuable sugge. In K. W. Spence & J. T. B. T.-P.

of L. and M. Spence (Eds.), (Vol. 2, pp. 89–195). Academic Press. doi:https://doi.org/10.1016/S0079-7421(08)60422-3

Awh, E., Jonides, J., Smith, E. E., Buxton, R. B., Frank, L. R., Love, T., … Gmeindl, L. (1999). Rehearsal in spatial working memory: Evidence from neuroimaging. *Psychological Science*, *10*(5), 433–437.

Baddeley, A.D., & Hitch, G. J. (1974). Working Memory. In G. A. Bower (Ed.), *The Psychology of Learning and Motivation* (pp. 47–89). Academic Press.

Baddeley, A. (1992). Working memory. *Science*, *255*(5044), 556 LP – 559. Retrieved from http://science.sciencemag.org/content/255/5044/556.abstract

Baddeley, A D, Hitch, G. J., & Allen, R. J. (2009). Working memory and binding in sentence recall. *Journal of Memory and Language*, *61*(3), 438–456. doi:https://doi.org/10.1016/j.jml.2009.05.004

Baddeley, A D, & Lieberman, K. (1980). Spatial working memory and imagery mneumonics. *Attention and Performance VIII. Englewood Cliffs*, *51*.

Baddeley, Alan. (1986). Working memory (vol. 11). Oxford: Oxford University Press, Clarendon Press.

Baddeley, Alan. (1996). The fractionation of working memory. *Proceedings of the National Academy of Sciences*, *93*(24), 13468 LP – 13472. doi:10.1073/pnas.93.24.13468

Baddeley, Alan. (2000). The episodic buffer: a new component of working memory? *Trends in Cognitive Sciences*, *4*(11), 417–423. doi:https://doi.org/10.1016/S1364-6613(00)01538-2

Baddeley, Alan. (2010). Working memory. *Current Biology*, *20*(4), R136–R140. doi:https://doi.org/10.1016/j.cub.2009.12.014

Baddeley, Alan. (2011). Working Memory: Theories, Models, and Controversies. *Annual Review of Psychology*, *63*(1), 1–29. doi:10.1146/annurev-psych-120710-100422

Baddeley, Alan D, Thomson, N., & Buchanan, M. (1975). Word length and the structure of short-term memory. *Journal of Verbal Learning and Verbal Behavior*, *14*(6), 575–589. doi:https://doi.org/10.1016/S0022-5371(75)80045-4

Baddeley, Alan, Della Sala, S., Papagno, C., & Spinnler, H. (1997). Dual-task performance in dysexecutive and nondysexecutive patients with a frontal lesion. *Neuropsychology*, *11*(2), 187.

Baddeley, Alan, Gathercole, S., & Papagno, C. (1998). The phonological loop as a language learning device. *Psychological Review*, *105*(1), 158.

Baddeley, Alan, Lewis, V., & Vallar, G. (1984). Exploring the articulatory loop. *The Quarterly Journal of Experimental Psychology Section A*, *36*(2), 233–252.

Baddeley, Alan, Papagno, C., & Vallar, G. (1988). When long-term learning depends on short-term storage. *Journal of Memory and Language*, *27*(5), 586–595. doi:https://doi.org/10.1016/0749-596X(88)90028-9

Baddeley, Alan, Vallar, G., & Wilson, B. (1987). Sentence comprehension and phonological memory: Some neuropsychological evidence.

Baddeley, Alan, & Wilson, B. (1988). Frontal amnesia and the dysexecutive syndrome. *Brain and Cognition*, *7*(2), 212–230. doi:https://doi.org/10.1016/0278-2626(88)90031-0

Baddeley, Alan, & Wilson, B. A. (2002). Prose recall and amnesia: implications for the structure of working memory. *Neuropsychologia*, *40*(10), 1737–1743. doi:https://doi.org/10.1016/S0028-3932(01)00146-4

Baltes, P. B., Reese, H. W., & Nesselroade, J. R. (2014). *Life-span developmental psychology: Introduction to research methods*. Psychology Press.

Baroody, A. J. (1984). More precisely defining and measuring the order-irrelevance principle. *Journal of Experimental Child Psychology*, *38*(1), 33–41. doi:https://doi.org/10.1016/0022-0965(84)90017-1

Baroody, A. J. (1992). The development of kindergartners' mental-addition strategies. *Learning and Individual Differences*, *4*(3), 215–235. doi:https://doi.org/10.1016/1041-6080(92)90003-W

Baroody, A. J. (1993). The Relationship between the Order-Irrelevance Principle and Counting Skill. *Journal for Research in Mathematics Education*, *24*(5), 415–427. doi:10.2307/749151

Baroody, A. J., & Price, J. (1983). The development of the number-word sequence in the counting of three-year-olds. *Journal for Research in Mathematics Education*, *14*(5), 361–368.

Barrouillet, P., Bernardin, S., & Camos, V. (2004). Time constraints and resource sharing in adults' working memory spans. *Journal of Experimental Psychology: General*, *133*(1), 83.

Barrouillet, P., Bernardin, S., Portrat, S., Vergauwe, E., & Camos, V. (2007). Time and cognitive load in working memory. *Journal of Experimental Psychology: Learning, Memory, and Cognition*. Barrouillet, Pierre: Universite de Geneve, 40 boulevard du Pont d'Arve, Geneva, Switzerland, Pierre.Barrouillet@pse.unige.ch: American Psychological Association. doi:10.1037/0278-7393.33.3.570

Barry, A. E. (2005). How attrition impacts the internal and external validity of longitudinal research. *The Journal of School Health*, *75*(7), 267.

Bayliss, D. M., Jarrold, C., Baddeley, A. D., & Gunn, D. M. (2005). The relationship between short-term memory and working memory: Complex span made simple? *MEMORY*, *13*(3–4), 414–421. doi:10.1080/09658210344000332

Benjamini, Y., & Hochberg, Y. (1995). Controlling the false discovery rate: a practical and powerful approach to multiple testing. *Journal of the Royal Statistical Society: Series B (Methodological)*, *57*(1), 289–300.

Bennett, J., & Müller, U. (2010). The Development of Flexibility and Abstraction in Preschool Children. *Merrill-Palmer Quarterly*, *56*(4), 455–473. Retrieved from http://www.jstor.org/stable/23097951

Benoit, L., Lehalle, H., & Jouen, F. (2004). Do young children acquire number words through subitizing or counting? *Cognitive Development*, *19*(3), 291–307. doi:https://doi.org/10.1016/j.cogdev.2004.03.005

BERA. (n.d.). Ethical Guidelines for Educational Research - 4th edition. Retrieved from https://www.bera.ac.uk/publication/ethical-guidelines-for-educational-research-2018-online

Berch, D. B. (2005). Making Sense of Number Sense: Implications for Children With Mathematical Disabilities. *Journal of Learning Disabilities*, *38*(4), 333–339. doi:10.1177/00222194050380040901

Berch, D. B., Krikorian, R., & Huha, E. M. (1998). The Corsi block-tapping task: Methodological and theoretical considerations. *Brain and Cognition*, *38*(3), 317–338.

Bermejo, V. (1996). Cardinality development and counting. *Developmental Psychology*, *32*(2), 263.

Bizzaro, M., Giofrè, D., Girelli, L., & Cornoldi, C. (2018). Arithmetic, working memory, and visuospatial imagery abilities in children with poor geometric learning. *Learning and Individual Differences*, *62*, 79–88.

Blalock, L. D., & Clegg, B. A. (2010). Encoding and representation of simultaneous and sequential arrays in visuospatial working memory. *Quarterly Journal of Experimental Psychology*, *63*(5), 856–862.

Bonny, J. W., & Lourenco, S. F. (2013). The approximate number system and its relation to early math achievement: evidence from the preschool years. *Journal of Experimental Child Psychology*, *114*(3), 375–388. doi:10.1016/j.jecp.2012.09.015

Booth, R. D. L., & Thomas, M. O. J. (1999). Visualization in mathematics learning: Arithmetic problem-solving and student difficulties. *The Journal of Mathematical Behavior*, *18*(2), 169–190.

Borenstein, M., Hedges, L., Higgins, J., & Rothstein, H. (2009). Complex data structures. *Introduction to Meta-Analysis. John Wiley & Sons Ltd., West Sussex, UK*, 215–245.

Bornstein, M. H., & Haynes, O. M. (1998). Vocabulary Competence in Early Childhood: Measurement, Latent Construct, and Predictive Validity. *Child Development*, *69*(3), 654–671. doi:10.1111/j.1467-8624.1998.tb06235.x

Bowen, D. J., Kreuter, M., Spring, B., Cofta-Woerpel, L., Linnan, L., Weiner, D., … Fernandez, M. (2009). How we design feasibility studies. *American Journal of Preventive Medicine*, *36*(5), 452–457. doi:10.1016/j.amepre.2009.02.002

Boyer, C. B., & Merzbach, U. C. (2011). *A History of Mathematics*. Wiley. Retrieved from https://books.google.co.uk/books?id=bR9HAAAAQBAJ

BPS. (n.d.). BPS Code of Ethics and Conduct. Retrieved from https://www.bps.org.uk/news-and-policy/bps-code-ethics-and-conduct

Brannon, E. M. (2002). The development of ordinal numerical knowledge in infancy. *Cognition*, *83*(3), 223–240.

Bresgi, L., Alexander, D. L. M., & Seabi, J. (2017). The predictive relationships between working memory skills within the spatial and verbal domains and mathematical performance of Grade 2 South African learners. *International Journal of Educational Research*, *81*(Supplement C), 1–10. doi:https://doi.org/10.1016/j.ijer.2016.10.004

Briars, D., & Siegler, R. S. (1984). A featural analysis of preschoolers' counting knowledge. *Developmental Psychology*, *20*(4), 607.

Broadbent, D. E. (1975). The magic number seven after fifteen years. *Studies in Long Term Memory*, 3–18.

Bromwich, D., & Rid, A. (2015). Can informed consent to research be adapted to risk? *Journal of Medical Ethics*, *41*(7), 521 LP – 528. doi:10.1136/medethics-2013-101912

Bruner, J. S. (1960). The process of education. Cambridge: MA Harvard University Press.

Bull, R, Johnston, R. S., & Roy, J. A. (1999). Exploring the roles of the visual-spatial sketch pad and central executive in children's arithmetical skills: Views from cognition and developmental neuropsychology. *Developmental Neuropsychology*, *15*, 421–442.

Bull, Rebecca, Espy, K. A., & Wiebe, S. A. (2008). Short-Term Memory, Working Memory, and Executive Functioning in Preschoolers: Longitudinal Predictors of Mathematical Achievement at Age 7 Years. *Developmental Neuropsychology*, *33*(3), 205–228. doi:10.1080/87565640801982312

Bull, Rebecca, & Johnston, R. S. (1997). Children's Arithmetical Difficulties: Contributions from Processing Speed, Item Identification, and Short-Term Memory. *Journal of Experimental Child Psychology*, *65*(1), 1–24. doi:https://doi.org/10.1006/jecp.1996.2358

Bunting, M., Cowan, N., & Scott Saults, J. (2006). How does running memory span work? *Quarterly Journal of Experimental Psychology*, *59*(10), 1691–1700.

Butterworth, B. (1999). *The mathematical brain*. Macmillan.

Butterworth, B. (2005). The development of arithmetical abilities. *Journal of Child Psychology and Psychiatry*, *46*(1), 3–18. doi:10.1111/j.1469-7610.2004.00374.x

Button, K. S., Ioannidis, J. P. A., Mokrysz, C., Nosek, B. A., Flint, J., Robinson, E. S. J., & Munafò, M. R. (2013). Power failure: why small sample size undermines the reliability of neuroscience. *Nature Reviews Neuroscience*, *14*, 365. Retrieved from http://dx.doi.org/10.1038/nrn3475

Cain, K., Oakhill, J., & Bryant, P. (2004). Children's reading comprehension ability: Concurrent prediction by working memory, verbal ability, and component skills. *Journal of Educational Psychology*, *96*(1), 31.

Camos, V., Mora, G., & Oberauer, K. (2011). Adaptive choice between articulatory rehearsal and attentional refreshing in verbal working memory. *Memory & Cognition*, *39*(2), 231–244. doi:10.3758/s13421-010-0011-x

Campos, I. S., Almeida, L. S., Ferreira, A. I., Martinez, L. F., & Ramalho, G. (2013). Cognitive processes and math performance: a study with children at third grade of basic education. *European Journal of Psychology of Education*, *28*(2), 421–436. Retrieved from http://www.jstor.org/stable/23421902

Carey, E., Hill, F., Devine, A., & Szücs, D. (2016). The Chicken or the Egg? The Direction of the Relationship Between Mathematics Anxiety and Mathematics Performance . *Frontiers in Psychology* . Retrieved from https://www.frontiersin.org/article/10.3389/fpsyg.2015.01987

Carey, S. (2001). On the very possibility of discontinuities in conceptual development. *Language, Brain, and Cognitive Development: Essays in Honor of Jacques Mehler*, 303–324.

Carlesimo, G. A., Perri, R., Turriziani, P., Tomaiuolo, F., & Caltagirone, C. (2001). Remembering What But Not Where: Independence of Spatial and Visual Working Memory in the Human Brain. *Cortex*, *37*(4), 519–534. doi:https://doi.org/10.1016/S0010-9452(08)70591-4

Carretti, B., Cornoldi, C., De Beni, R., & Palladino, P. (2004). What happens to information to be suppressed in working-memory tasks? Short and long term effects. *The Quarterly Journal of Experimental Psychology.*, *57*(6), 1059–84. doi:10.1080/02724980343000684

Case, R., Kurland, D. M., & Goldberg, J. (1982). Operational efficiency and the growth of short-term memory span. *Journal of Experimental Child Psychology*, *33*(3), 386–404.

Caviola, S., Colling, L. J., Mammarella, I. C., & Szűcs, D. (2020). Predictors of mathematics in primary school: magnitude comparison, verbal and spatial working memory measures. *Developmental Science*, e12957.

Caviola, S., Gerotto, G., & Mammarella, I. C. (2016). Computer-based training for improving mental calculation in third-and fifth-graders. *Acta Psychologica*, *171*, 118–127.

Caviola, S., Mammarella, I. C., Cornoldi, C., & Lucangeli, D. (2012). The involvement of working memory in children's exact and approximate mental addition. *JOURNAL OF EXPERIMENTAL CHILD PSYCHOLOGY*, *112*(2), 141–160. doi:10.1016/j.jecp.2012.02.005

Caviola, S., Mammarella, I. C., Lucangeli, D., & Cornoldi, C. (2014). Working memory and domain-specific precursors predicting success in learning written subtraction problems. *LEARNING AND INDIVIDUAL DIFFERENCES*, *36*, 92–100. doi:10.1016/j.lindif.2014.10.010

Caviola, S., Mammarella, I. C., Pastore, M., & LeFevre, J.-A. (2018). Children's strategy choices on complex subtraction problems: individual differences and developmental changes. *Frontiers in Psychology*, *9*, 1209.

Chan, L. K. S., Cole, P. G., & Morris, J. N. (1990). Effects of Instruction in the Use of a Visual-Imagery Strategy on the Reading-Comprehension Competence of Disabled and Average Readers. *Learning Disability Quarterly*, *13*(1), 2–11. doi:10.2307/1510388

Chard, D. J., Clarke, B., Baker, S., Otterstedt, J., Braun, D., & Katz, R. (2005). Using Measures of Number Sense to Screen for Difficulties in Mathematics: Preliminary Findings. *Assessment for Effective Intervention*, *30*(2), 3–14. doi:10.1177/073724770503000202

Chassang, G. (2017). The impact of the EU general data protection regulation on scientific research. *Ecancermedicalscience*, *11*, 709. doi:10.3332/ecancer.2017.709

Chen, H. (2018). VennDiagram: Generate High-Resolution Venn and Euler Plots. Retrieved from https://cran.r-project.org/package=VennDiagram

Chen, Z., & Cowan, N. (2005). Chunk limits and length limits in immediate recall: a reconciliation. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *31*(6), 1235.

Cheung, A. C. K., & Slavin, R. E. (2016). How methodological features affect effect sizes in education. *Educational Researcher*, *45*(5), 283–292.

Chu, F. W., vanMarle, K., & Geary, D. C. (2016). Predicting Children's Reading and Mathematics Achievement from Early Quantitative Knowledge and Domain-General Cognitive Abilities . *Frontiers in Psychology* . Retrieved from https://www.frontiersin.org/article/10.3389/fpsyg.2016.00775

Chuah, Y. M., & Maybery, M. T. (1999). Verbal and spatial short-term memory: common sources of developmental change? *Journal of Experimental Child Psychology*, *73*(1), 7–44. doi:10.1006/jecp.1999.2493

Clearman, J., Klinger, V., & Szucs, D. (2017). Visuospatial and verbal memory in mental arithmetic. *QUARTERLY JOURNAL OF EXPERIMENTAL PSYCHOLOGY*, *70*(9), 1837–1855. doi:10.1080/17470218.2016.1209534

Clements, D. H. (1999). Subitizing: What is it? Why teach it? *Teaching Children Mathematics*, *5*(7), 400.

Coe, R., Waring, M., Hedges, L., & Arthur, J. (Eds.). (2017). *Research methods and methodologies in education*. Thousand Oaks, CA: Sage.

Cohen, J., Cohen, P., West, S. G., & Aiken, L. S. (2013). *Applied multiple regression/correlation analysis for the behavioral sciences*. Routledge.

Colom, R., Abad, F. J., Rebollo, I., & Chun Shih, P. (2005). Memory span and general intelligence: A latent-variable approach. *Intelligence*, *33*(6), 623–642. doi:https://doi.org/10.1016/j.intell.2005.05.006

CONRAD, R., & HULL, A. J. (1964). INFORMATION, ACOUSTIC CONFUSION AND MEMORY SPAN. *British Journal of Psychology*, *55*(4), 429–432. doi:10.1111/j.2044-8295.1964.tb00928.x

Conway, A. R. A., Cowan, N., Bunting, M. F., Therriault, D. J., & Minkoff, S. R. B. (2002). A latent variable analysis of working memory capacity, short-term memory capacity, processing speed, and general fluid intelligence. *Intelligence*, *30*(2), 163–183. doi:https://doi.org/10.1016/S0160-2896(01)00096-4

Conway, A. R. A., Kane, M. J., Bunting, M. F., Hambrick, D. Z., Wilhelm, O., & Engle, R. W. (2005). Working memory span tasks: A methodological review and user's guide. *Psychonomic Bulletin & Review*, *12*(5), 769–786.

Conway, A. R. A., Kane, M. J., & Engle, R. W. (2003). Working memory capacity and its relation to general intelligence. *Trends in Cognitive Sciences*, *7*(12), 547–552. doi:https://doi.org/10.1016/j.tics.2003.10.005

Cooper, H. (2016). Ethical choices in research: Managing data, writing reports, and publishing results in the social sciences. *Ethical Choices in Research: Managing Data, Writing Reports, and Publishing Results in the Social Sciences.* Washington, DC, US: American Psychological Association. doi:10.1037/14859-000

Cornoldi, C., & Giofrè, D. (2014). The Crucial Role of Working Memory in Intellectual Functioning. *European Psychologist*, *19*(4), 260–268. doi:10.1027/1016-9040/a000183

Cornoldi, C., Giofrè, D., Calgaro, G., & Stupiggia, C. (2013). Attentional WM is not necessarily specifically related with fluid intelligence: The case of smart children with ADHD symptoms. *Psychological Research*, *77*(4), 508–515.

Cornoldi, C., Orsini, A., Cianci, L., Giofrè, D., & Pezzuti, L. (2013). Intelligence and working memory control: Evidence from the WISC-IV administration to Italian children. *Learning and Individual Differences*, *26*, 9–14.

Cornoldi, C., & Vecchi, T. (2000). Mental imagery in blind people: the role of passive and active visuospatial processes. In *Touch, Representation, and Blindness* (pp. 142–181). Oxford University Press. doi:10.1093/acprof:oso/9780198503873.003.0005

Cornoldi, C., & Vecchi, T. (2004). *Visuo-spatial Working Memory and Individual Differences*. Psychology Press. doi:10.4324/9780203641583

Corsi, P. M. (1972). *Memory and the medial temporal region of the brain*. McGill University.

Costantini, G., Richetin, J., Borsboom, D., Fried, E. I., Rhemtulla, M., & Perugini, M. (2015). Development of indirect measures of conscientiousness: Combining a facets approach and network analysis. *European Journal of Personality*, *29*(5), 548–567.

Cowan, N. (1988). Evolving conceptions of memory storage, selective attention, and their mutual constraints within the human information-processing system. *Psychological Bulletin*, *104*(2), 163.

Cowan, N. (1992). Verbal Memory Span and the Timing of Spoken Recall. *Journal of Memory and Language*, *31*(5), 668.

Cowan, N. (1999). An embedded-processes model of working memory. *Models of Working Memory: Mechanisms of Active Maintenance and Executive Control*, *20*, 506.

Cowan, N. (2001). The magical number 4 in short-term memory: A reconsideration of mental storage capacity. *Behavioral and Brain Sciences*, *24*(1), 87–114.

Cowan, N. (2005). Working memory capacity limits in a theoretical context. In *Human learning and memory: Advances in theory and application. The 4th Tsukuba international conference on memory* (pp. 155–175).

Cowan, N. (2008). Chapter 20 What are the differences between long-term, short-term, and working memory? In W. S. Sossin, J.-C. Lacaille, V. F. Castellucci, & S. B. T.-P. in B. R. Belleville (Eds.), *Essence of Memory* (Vol. 169, pp. 323–338). Elsevier. doi:https://doi.org/10.1016/S0079-6123(07)00020-9

Cowan, N. (2010). The magical mystery four: How is working memory capacity limited, and why? *Current Directions in Psychological Science*, *19*(1), 51–57.

Cowan, N. (2016). *Working memory capacity: Classic edition*. Psychology press.

Cowan, N. (2017). The many faces of working memory and short-term storage. *Psychonomic Bulletin & Review*, *24*(4), 1158–1170. doi:10.3758/s13423-016-1191-6

Cowan, N., Elliott, E. M., Scott Saults, J., Morey, C. C., Mattox, S., Hismjatullina, A., & Conway, A. R. A. (2005). On the capacity of attention: Its estimation and its role in working memory and cognitive aptitudes. *Cognitive Psychology*, *51*(1), 42–100. doi:https://doi.org/10.1016/j.cogpsych.2004.12.001

Cowan, N., Li, Y., Glass, B. A., & Scott Saults, J. (2018). Development of the ability to combine visual and acoustic information in working memory. *Developmental Science*, *21*(5), e12635. doi:10.1111/desc.12635

Cowan, R., Dowker, A., Christakis, A., & Bailey, S. (1996). Even More Precisely Assessing Children's Understanding of the Order-Irrelevance Principle. *Journal of Experimental*

Child Psychology, *62*(1), 84–101. doi:https://doi.org/10.1006/jecp.1996.0023

Cragg, L., & Gilmore, C. (2014). Skills underlying mathematics: The role of executive function in the development of mathematics proficiency. *Trends in Neuroscience and Education*, *3*(2), 63–68. doi:https://doi.org/10.1016/j.tine.2013.12.001

Crawford, D. B., & Snider, V. E. (2000). Effective mathematics instruction the importance of curriculum. *Education and Treatment of Children*, 122–142.

D'Amico, A., & Guarnera, M. (2005). Exploring working memory in children with low arithmetical achievement. *Learning and Individual Differences*, *15*(3), 189–202. doi:https://doi.org/10.1016/j.lindif.2005.01.002

Daneman, M., & Carpenter, P. A. (1980). Individual differences in working memory and reading. *Journal of Verbal Learning and Verbal Behavior*, *19*(4), 450–466. doi:https://doi.org/10.1016/S0022-5371(80)90312-6

Dark, V. J., & Benbow, C. P. (1994). Type of stimulus mediates the relationship between working-memory performance and type of precocity. *Intelligence*, *19*(3), 337–357. doi:https://doi.org/10.1016/0160-2896(94)90006-X

Darling, S, Della Sala, S., Logie, R. H., & Cantagallo, A. (2006). Neuropsychological evidence for separating components of visuo–spatial working memory. *Journal of Neurology*, *253*(2), 176–180. doi:10.1007/s00415-005-0944-3

Darling, Stephen, Della Sala, S., & Logie, R. H. (2007). Behavioural evidence for separating components within visuo-spatial working memory. *Cognitive Processing*, *8*(3), 175–181. doi:10.1007/s10339-007-0171-1

David, C. V. (2012). Working memory deficits in math learning difficulties: A meta-analysis. *International Journal of Developmental Disabilities*, *58*(2), 67–84.

De Cruz, H. (2006). Towards a Darwinian Approach to Mathematics. *Foundations of Science*, *11*(1), 157–196. doi:10.1007/s10699-004-5916-z

De Smedt, B., & Boets, B. (2010). Phonological processing and arithmetic fact retrieval: Evidence from developmental dyslexia. *Neuropsychologia*, *48*(14), 3973–3981. doi:https://doi.org/10.1016/j.neuropsychologia.2010.10.018

De Smedt, B., Janssen, R., Bouwens, K., Verschaffel, L., Boets, B., & Ghesquière, P. (2009). Working memory and individual differences in mathematics achievement: A longitudinal study from first grade to second grade. *Journal of Experimental Child Psychology*, *103*(2), 186–201. doi:https://doi.org/10.1016/j.jecp.2009.01.004

DECKER, S. L., HILL, S. K., & DEAN, R. S. (2007). EVIDENCE OF CONSTRUCT SIMILARITY IN EXECUTIVE FUNCTIONS AND FLUID REASONING ABILITIES. *International Journal of Neuroscience*, *117*(6), 735–748. doi:10.1080/00207450600910085

Dehaene, S, Spelke, E., Pinel, P., Stanescu, R., & Tsivkin, S. (1999). Sources of Mathematical Thinking: Behavioral and Brain-Imaging Evidence. *Science*, *284*(5416), 970 LP – 974. doi:10.1126/science.284.5416.970

Dehaene, Stanislas. (1992). Varieties of numerical abilities. *Cognition*, *44*(1), 1–42. doi:https://doi.org/10.1016/0010-0277(92)90049-N

Dehaene, Stanislas. (2001). Précis of The Number Sense. *Mind & Language*, *16*(1), 16–36. doi:10.1111/1468-0017.00154

Dehaene, Stanislas. (2003). The neural basis of the Weber-Fechner law: A logarithmic mental number line. *Trends in Cognitive Sciences*. Netherlands: Elsevier Science. doi:10.1016/S1364-6613(03)00055-X

Dehaene, Stanislas. (2011). *The number sense: How the mind creates mathematics*. OUP USA.

Dehaene, Stanislas, & Cohen, L. (1994). Dissociable mechanisms of subitizing and counting: neuropsychological evidence from simultanagnosic patients. *Journal of Experimental Psychology: Human Perception and Performance*, *20*(5), 958.

Dehaene, Stanislas, & Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical Cognition*, *1*(1), 83–120.

Dehaene, Stanislas, Dupoux, E., & Mehler, J. (1990). Is numerical comparison digital? Analogical and symbolic effects in two-digit number comparison. *Journal of Experimental Psychology: Human Perception and Performance*, *16*(3), 626.

Della Sala, S., Gray, C., Baddeley, A., Allamano, N., & Wilson, L. (1999). Pattern span: a tool for unwelding visuo–spatial memory. *Neuropsychologia*, *37*(10), 1189–1199.

Deng, L., & Chan, W. (2016). Testing the Difference Between Reliability Coefficients Alpha and Omega. *Educational and Psychological Measurement*, *77*(2), 185–203. doi:10.1177/0013164416658325

Department for Education. (2013). Mathematics programmes of study: key stages 1 and 2. *Department for Education*, (September), 1–5. doi:10.1017/CBO9781107415324.004

Devine, A., Soltész, F., Nobes, A., Goswami, U., & Szűcs, D. (2013). Gender differences in developmental dyscalculia depend on diagnostic criteria. *Learning and Instruction*, *27*, 31–39.

Donolato, E., Giofrè, D., & Mammarella, I. C. (2017). Differences in verbal and visuospatial forward and backward order recall: A review of the literature. *Frontiers in Psychology*, *8*, 663.

Dove, E. S. (2018). The EU General Data Protection Regulation: Implications for International Scientific Research in the Digital Era. *The Journal of Law, Medicine & Ethics*, *46*(4), 1013–1030. doi:10.1177/1073110518822003

Dowding, T. J. (1993). The application of a spiral curriculum model to technical training curricula. *Educational Technology*, *33*(7), 18–28.

Dunning, D. L., Holmes, J., & Gathercole, S. E. (2013). Does working memory training lead to generalized improvements in children with low working memory? A randomized controlled trial. *Developmental Science*, *16*(6), 915–925. doi:10.1111/desc.12068

Einarsdóttir, J. (2007). Research with children: methodological and ethical challenges. *European Early Childhood Education Research Journal*, *15*(2), 197–211. doi:10.1080/13502930701321477

ELLIOTT, J. G., & GIBBS, S. (2008). Does Dyslexia Exist? *Journal of Philosophy of Education*, *42*(3-4), 475–491. doi:10.1111/j.1467-9752.2008.00653.x

Elliott, J. G., & Grigorenko, E. L. (2014). *The dyslexia debate*. Cambridge University Press.

Engle, R. W. (2002). Working memory capacity as executive attention. *Current Directions in Psychological Science*, *11*(1), 19–23.

Engle, R. W. (2010). Role of working-memory capacity in cognitive control. *Current Anthropology*, *51*(S1), S17–S26.

Engle, R. W., Cantor, J., & Carullo, J. J. (1992). Individual differences in working memory and comprehension: A test of four hypotheses. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *18*(5), 972.

Engle, R. W., Tuholski, S. W., Laughlin, J. E., & Conway, A. R. A. (1999). Working memory, short-term memory, and general fluid intelligence: a latent-variable approach. *Journal of Experimental Psychology: General*, *128*(3), 309.

Epskamp, S., & Fried, E. I. (2018). A tutorial on regularized partial correlation networks. *Psychological Methods*, *23*(4), 617.

Fabbri, S., Caviola, S., Tang, J., Zorzi, M., & Butterworth, B. (2012). The role of numerosity in processing nonsymbolic proportions. *The Quarterly Journal of Experimental Psychology*, *65*(12), 2435–2446.

Fanari, R., Meloni, C., & Massidda, D. (2019). Visual and Spatial Working Memory Abilities Predict Early Math Skills: A Longitudinal Study . *Frontiers in Psychology* . Retrieved from https://www.frontiersin.org/article/10.3389/fpsyg.2019.02460

Feigenson, L., & Carey, S. (2005). On the limits of infants' quantification of small object arrays. *Cognition*, *97*(3), 295–313. doi:https://doi.org/10.1016/j.cognition.2004.09.010

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences*, *8*(7), 307–314. doi:https://doi.org/10.1016/j.tics.2004.05.002

Feigenson, L., Libertus, M. E., & Halberda, J. (2013). Links Between the Intuitive Sense of Number and Formal Mathematics Ability. *Child Development Perspectives*, *7*(2), 74–79. doi:10.1111/cdep.12019

Flavell, J. H., Beach, D. R., & Chinsky, J. M. (1966). Spontaneous verbal rehearsal in a memory task as a function of age. *Child Development*, 283–299.

Fletcher, J. M. (1985). Memory for verbal and nonverbal stimuli in learning disability subgroups: Analysis by selective reminding. *Journal of Experimental Child Psychology*, *40*(2), 244–259.

FLOYD, R. G., EVANS, J. J., & McGREW, K. S. (2003). Relations between measures of Cattell-Horn-Carroll (CHC) cognitive abilities and mathematics achievement across the school-age years. *Psychology in the Schools*, *40*(2), 155–171. doi:10.1002/pits.10083

Fluck, M. J. (1995). Counting on the right number: Maternal support for the development of cardinality. *The Irish Journal of Psychology*, *16*(2), 133–149. doi:10.1080/03033910.1995.10558050

Foote, M. Q., & Bartell, T. G. (2011). Pathways to equity in mathematics education: How life experiences impact researcher positionality. *Educational Studies in Mathematics*, *78*(1), 45–68.

Freeman, N. H., Antonucci, C., & Lewis, C. (2000). Representation of the cardinality principle: early conception of error in a counterfactual test. *Cognition*, *74*(1), 71–89. doi:https://doi.org/10.1016/S0010-0277(99)00064-5

Friso-van den Bos, I., van der Ven, S. H. G., Kroesbergen, E. H., & van Luit, J. E. H. (2013). Working memory and mathematics in primary school children: A meta-analysis. *Educational Research Review*, *10*, 29–44. doi:https://doi.org/10.1016/j.edurev.2013.05.003

Fry, A. F., & Hale, S. (1996). Processing Speed, Working Memory, and Fluid Intelligence: Evidence for a Developmental Cascade. *Psychological Science*, *7*(4), 237–241. doi:10.1111/j.1467-9280.1996.tb00366.x

Fry, A. F., & Hale, S. (2000). Relationships among processing speed, working memory, and fluid intelligence in children. *Biological Psychology*, *54*(1), 1–34. doi:https://doi.org/10.1016/S0301-0511(00)00051-X

Frye, D., Braisby, N., Lowe, J., Maroudas, C., & Nicholls, J. (1989). Young Children's Understanding of Counting and Cardinality. *Child Development*, *60*(5), 1158–1171. doi:10.2307/1130790

Fuchs, L. S., Compton, D. L., Fuchs, D., Hollenbeck, K. N., Hamlett, C. L., & Seethaler, P. M. (2011). Two-Stage Screening for Math Problem-Solving Difficulty Using Dynamic Assessment of Algebraic Learning. *Journal of Learning Disabilities*, *44*(4), 372–380. doi:10.1177/0022219411407867

Fuchs, L. S., Fuchs, D., Compton, D. L., Powell, S. R., Seethaler, P. M., Capizzi, A. M., … Fletcher, J. M. (2006). The cognitive correlates of third-grade skill in arithmetic, algorithmic computation, and arithmetic word problems. *Journal of Educational Psychology*, *98*(1), 29.

Fuson, K. C., & Hall, J. W. (1983). The acquisition of early number word meaning: A conceptual analysis and review. *The Development of Mathematical Thinking. New York: Academic*.

Gallagher, M., Haywood, S. L., Jones, M. W., & Milne, S. (2010). Negotiating informed consent with children in school-based research: a critical review. *Children & Society*, *24*(6), 471–482.

Gallistel, C. R. (1990). *The organization of learning.* The MIT Press.

Gathercole, S., & Pickering, S. (2001). Working memory test battery for children. *London, UK: Psychological Corporation*.

Gathercole, S E, & Pickering, S. J. (2000). Working memory deficits in children with low achievements in the national curriculum at 7 years of age. *BRITISH JOURNAL OF EDUCATIONAL PSYCHOLOGY*, *70*(2), 177–194. doi:10.1348/000709900158047

Gathercole, Susan E. (1995). Is nonword repetition a test of phonological memory or long-term knowledge? It all depends on the nonwords. *Memory & Cognition*, *23*(1), 83–94.

Gathercole, Susan E. (1998). The development of memory. *The Journal of Child Psychology and Psychiatry and Allied Disciplines*, *39*(1), 3–27.

Gathercole, Susan E, & Alloway, T. P. (2004). Working memory and classroom learning. *Dyslexia Review*, *15*, 4–9.

Gathercole, Susan E, & Baddeley, A. D. (1993). Phonological working memory: A critical building block for reading development and vocabulary acquisition? *European Journal of Psychology of Education*, *8*(3), 259. doi:10.1007/BF03174081

Gathercole, Susan E, Brown, L., & Pickering, S. J. (2003). Working memory assessments at school entry as longitudinal predictors of National Curriculum attainment levels. *Educational and Child Psychology*, *20*(3), 109–122.

Gathercole, Susan E, Pickering, S. J., Ambridge, B., & Wearing, H. (2004). The Structure of Working Memory From 4 to 15 Years of Age. *Developmental Psychology*. Gathercole, Susan E.: Department of Psychology, University of Durham, Science Laboratories, South Road, Durham, England, DH1 3LE: American Psychological Association. doi:10.1037/0012-1649.40.2.177

Gathercole, Susan E, Pickering, S. J., Knight, C., & Stegmann, Z. (2004). Working memory skills and educational attainment: evidence from national curriculum assessments at 7 and 14 years of age. *Applied Cognitive Psychology*, *18*(1), 1–16. doi:10.1002/acp.934

Geary, D. C. (2011). Cognitive Predictors of Achievement Growth in Mathematics: A 5-Year Longitudinal Study. *DEVELOPMENTAL PSYCHOLOGY*, *47*(6), 1539–1552. doi:10.1037/a0025510

Geary, D. C., Bailey, D. H., & Hoard, M. K. (2009). Predicting Mathematical Achievement and Mathematical Learning Disability with a Simple Screening Tool: The Number Sets Test. *Journal of Psychoeducational Assessment*, *27*(3), 265–279. doi:10.1177/0734282908330592

Geary, D. C., Hoard, M. K., Byrd-Craven, J., & DeSoto, C. M. (2004). Strategy choices in simple and complex addition: Contributions of working memory and counting knowledge for children with mathematical disability. *Journal of Experimental Child Psychology*, *88*(2), 121–151. doi:https://doi.org/10.1016/j.jecp.2004.03.002

Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive Mechanisms Underlying Achievement Deficits in Children with Mathematical Learning Disability. *Child Development*, *78*(4), 1343–1359. Retrieved from http://www.jstor.org/stable/4620706

Geary, D. C., Nicholas, A., Li, Y., & Sun, J. (2017). Developmental change in the influence of domain-general abilities and domain-specific knowledge on mathematics achievement: an eight-year longitudinal study. *Journal of Educational Psychology*, *109*(5), 680.

GELMAN, R., & Gallistel, C. R. (1986). *The Child's Understanding of Number*. Harvard University Press. Retrieved from https://books.google.co.uk/books?id=95NJ6MxJcMQC

Gelman, R., & Meck, E. (1983). Preschoolers' counting: Principles before skill. *Cognition*, *13*(3), 343–359.

Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities*, *38*(4), 293–304.

Giaquinto, M. (2018). Cognitive access to numbers: the philosophical significance of empirical findings about basic number abilities. *Philosophical Transactions of the Royal Society B: Biological Sciences*, *373*(1740), 20160520.

Gibbs, B. C. (2014). Reconfiguring Bruner: Compressing the Spiral Curriculum. *Phi Delta*

Kappan, *95*(7), 41–44. doi:10.1177/003172171409500710

Gibbs, S., & Elliott, J. (2015). The differential effects of labelling: how do ‘dyslexia’ and ‘reading difficulties’ affect teachers’ beliefs. *European Journal of Special Needs Education*, *30*(3), 323–337. doi:10.1080/08856257.2015.1022999

Gilchrist, A. L., Cowan, N., & Naveh-Benjamin, M. (2009). Investigating the childhood development of working memory using sentences: New evidence for the growth of chunk capacity. *Journal of Experimental Child Psychology*, *104*(2), 252–265.

Gilmore, C., Attridge, N., Clayton, S., Cragg, L., Johnson, S., Marlow, N., … Inglis, M. (2013). Individual Differences in Inhibitory Control, Not Non-Verbal Number Acuity, Correlate with Mathematics Achievement. *PLOS ONE*, *8*(6), e67374. Retrieved from https://doi.org/10.1371/journal.pone.0067374

Gilmore, C., Attridge, N., & Inglis, M. (2011). Measuring the approximate number system. *The Quarterly Journal of Experimental Psychology*, *64*(11), 2099–2109. doi:10.1080/17470218.2011.574710

Ginsburg, N. (1978). Perceived Numerosity, Item Arrangement, and Expectancy. *The American Journal of Psychology*, *91*(2), 267–273. doi:10.2307/1421536

Giofrè, D., Borella, E., & Mammarella, I. C. (2017). The relationship between intelligence, working memory, academic self-esteem, and academic achievement. *Journal of Cognitive Psychology*, *29*(6), 731–747.

Giofrè, D., Donolato, E., & Mammarella, I. C. (2018a). The differential role of verbal and visuospatial working memory in mathematics and reading. *Trends in Neuroscience and Education*, *12*, 1–6. doi:https://doi.org/10.1016/j.tine.2018.07.001

Giofrè, D., Donolato, E., & Mammarella, I. C. (2018b). Verbal and visuospatial WM & academic achievement. *Trends in Neuroscience and Education*, *12*(July), 1–6. doi:10.1016/j.tine.2018.07.001

Giofrè, D., & Mammarella, I. C. (2014). The relationship between working memory and intelligence in children: Is the scoring procedure important? *Intelligence*, *46*, 300–310.

Giofrè, D., Mammarella, I. C., & Cornoldi, C. (2013). The structure of working memory and how it relates to intelligence in children. *Intelligence*, *41*(5), 396–406. doi:https://doi.org/10.1016/j.intell.2013.06.006

Giofrè, D., Mammarella, I. C., & Cornoldi, C. (2014a). The relationship among geometry, working memory, and intelligence in children. *Journal of Experimental Child Psychology*, *123*, 112–128. doi:10.1016/j.jecp.2014.01.002

Giofrè, D., Mammarella, I. C., & Cornoldi, C. (2014b). The relationship among geometry, working memory, and intelligence in children. *Journal of Experimental Child Psychology*, *123*, 112–128. doi:10.1016/J.JECP.2014.01.002

Giofrè, D., Mammarella, I. C., Ronconi, L., & Cornoldi, C. (2013). Visuospatial working memory in intuitive geometry, and in academic achievement in geometry. *Learning and Individual Differences*, *23*, 114–122. doi:10.1016/J.LINDIF.2012.09.012

Glanzer, M., & Razel, M. (1974). The size of the unit in short-term storage. *Journal of Verbal Learning and Verbal Behavior*, *13*(1), 114–131.

Gliga, F., & Gliga, T. (2012). Romanian screening instrument for dyscalculia. *Procedia - Social and Behavioral Sciences*, *33*, 15–19. doi:https://doi.org/10.1016/j.sbspro.2012.01.074

Göbel, S. M., Watson, S. E., Lervåg, A., & Hulme, C. (2014). Children’s arithmetic development: It is number knowledge, not the approximate number sense, that counts. *Psychological Science*, *25*(3), 789–798.

Godijn, R., & Theeuwes, J. (2012). Overt is no better than covert when rehearsing visuo-spatial information in working memory. *Memory & Cognition*, *40*(1), 52–61. doi:10.3758/s13421-011-0132-x

Goldstein, D. J., Allen, C. M., & Fleming, L. P. (1982). Relationship between the expressive one-word picture vocabulary test and measures of intelligence, receptive vocabulary, and visual-motor coordination in borderline and mildly retarded children. *Psychology in the Schools*, *19*(3), 315–318. doi:10.1002/1520-6807(198207)19:3<315::AID-PITS2310190309>3.0.CO;2-T

Goodman, J. S., & Blum, T. C. (1996). Assessing the non-random sampling effects of subject attrition in longitudinal research. *Journal of Management*, *22*(4), 627–652.

Graham, A., Powell, M. A., & Taylor, N. (2015). Ethical Research Involving Children: Encouraging Reflexive Engagement in Research with Children and Young People. *Children & Society*, *29*(5), 331–343. doi:10.1111/chso.12089

Grant, H. (1999). Mathematics and the liberal arts. *The College Mathematics Journal*, *30*(2), 96–105.

Greeno, J. G. (1991). Number sense as situated knowing in a conceptual domain. *Journal for Research in Mathematics Education*, 170–218.

Greeno, J. G., Riley, M. S., & Gelman, R. (1984). Conceptual competence and children’s counting. *Cognitive Psychology*, *16*(1), 94–143. doi:https://doi.org/10.1016/0010-0285(84)90005-7

Gross-Tsur, V., Manor, O., & Shalev, R. S. (1996). DEVELOPMENTAL DYSCALCULIA: PREVALENCE AND DEMOGRAPHIC FEATURES. *Developmental Medicine & Child Neurology*, *38*(1), 25–33. doi:10.1111/j.1469-8749.1996.tb15029.x

Guérard, K., & Tremblay, S. (2008). Revisiting evidence for modularity and functional equivalence across verbal and spatial domains in memory. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *34*(3), 556.

Guttentag, R. E. (1984). The mental effort requirement of cumulative rehearsal: A developmental study. *Journal of Experimental Child Psychology*, *37*(1), 92–106. doi:https://doi.org/10.1016/0022-0965(84)90060-2

Gyselinck, V., De Beni, R., Pazzaglia, F., Meneghetti, C., & Mondoloni, A. (2006). Working memory components and imagery instructions in the elaboration of a spatial mental model. *Psychological Research*, *71*(3), 373. doi:10.1007/s00426-006-0091-1

H. Logie, R., & Pearson, D. G. (1997). The inner eye and the inner scribe of visuo-spatial working memory: Evidence from developmental fractionation. *European Journal of Cognitive Psychology*, *9*(3), 241–257.

Haavisto, M.-L., & Lehto, J. E. (2005). Fluid/spatial and crystallized intelligence in relation to domain-specific working memory: A latent-variable approach. *Learning and Individual Differences*, *15*(1), 1–21. doi:https://doi.org/10.1016/j.lindif.2004.04.002

Hadley Wickham. (2016). *ggplot2: Elegant graphics for data analysis*. *Springer-Verlag New York*.

Halberda, J., Mazzocco, M. M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature*, *455*, 665. Retrieved from https://doi.org/10.1038/nature07246

Harden, R. M. (1999). What is a spiral curriculum? *Medical Teacher*, *21*(2), 141–143.

Heathcote, D. (1994). The role of visuo-spatial working memory in the mental addition of multi-digit addends. *Cahiers de Psychologie Cognitive/Current Psychology of Cognition*.

Hecht, S. A. (2002). Counting on working memory in simple arithmetic when counting is used for problem solving. *Memory & Cognition*, *30*(3), 447–455.

Henry, L. A., Messer, D., Luger-Klein, S., & Crane, L. (2012). Phonological, visual, and semantic coding strategies and children's short-term picture memory span. *The Quarterly Journal of Experimental Psychology*, *65*(10), 2033–2053. doi:10.1080/17470218.2012.672997

Henson, R. (2002). Working Memory in Perspective. In J. Andrade (Ed.), . Taylor & Francis. Retrieved from https://books.google.co.uk/books?id=Akd4AgAAQBAJ

Hilbert, S., Bruckmaier, G., Binder, K., Krauss, S., & Bühner, M. (2019). Prediction of elementary mathematics grades by cognitive abilities. *European Journal of Psychology of Education*, *34*(3), 665–683. doi:10.1007/s10212-018-0394-9

Hitch, G. J., & McAuley, E. (1991). Working memory in children with specific arithmetical learning difficulties. *British Journal of Psychology*, *82*(3), 375–386. doi:10.1111/j.2044-8295.1991.tb02406.x

Hoard, M. K., Geary, D. C., Byrd-Craven, J., & Nugent, L. (2008). Mathematical Cognition in Intellectually Precocious First Graders. *Developmental Neuropsychology*, *33*(3), 251–276. doi:10.1080/87565640801982338

Holmes, J., & Adams, J. W. (2006). Working Memory and Children's Mathematical Skills: Implications for mathematical development and mathematics curricula. *Educational Psychology*, *26*(3), 339–366. doi:10.1080/01443410500341056

Holmes, J., Adams, J. W., & Hamilton, C. J. (2008). The relationship between visuospatial sketchpad capacity and children's mathematical skills. *EUROPEAN JOURNAL OF COGNITIVE PSYCHOLOGY*, *20*(2), 272–289. doi:10.1080/09541440701612702

Holmes, J., & Gathercole, S. E. (2014). Taking working memory training from the laboratory into schools. *Educational Psychology*, *34*(4), 440–450. doi:10.1080/01443410.2013.797338

Hornung, C., Schiltz, C., Brunner, M., & Martin, R. (2014). Predicting first-grade mathematics achievement: the contributions of domain-general cognitive abilities, nonverbal number sense, and early number competence . *Frontiers in Psychology* . Retrieved from https://www.frontiersin.org/article/10.3389/fpsyg.2014.00272

Hu, L., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modeling: A Multidisciplinary Journal*, *6*(1), 1–55. doi:10.1080/10705519909540118

Huizinga, M., Dolan, C. V, & van der Molen, M. W. (2006). Age-related change in executive function: Developmental trends and a latent variable analysis. *Neuropsychologia*, *44*(11), 2017–2036.

Hulme, C., & Tordoff, V. (1989). Working memory development: The effects of speech rate, word length, and acoustic similarity on serial recall. *Journal of Experimental Child Psychology*, *47*(1), 72–87. doi:https://doi.org/10.1016/0022-0965(89)90063-5

Imbo, I., & Vandierendonck, A. (2007). Do multiplication and division strategies rely on executive and phonological working memory resources? *Memory & Cognition*, *35*(7), 1759–1771. doi:10.3758/BF03193508

Inglis, M., & Gilmore, C. (2014). Indexing the approximate number system. *Acta Psychologica*, *145*, 147–155. doi:https://doi.org/10.1016/j.actpsy.2013.11.009

Jarvis, H. L., & Gathercole, S. (2003). Verbal and non-verbal working memory and achievements on National Curriculum tests at 11 and 14 years of age. *Educational and Child Psychology*, *20*, 123–140. Retrieved from https://www.researchgate.net/profile/Lorna_Bourke/publication/250928054_The_relationship_between_working_memory_and_early_writing_at_the_word_sentence_and_text_level/links/02e7e51ee5262398b0000000.pdf#page=125

Jones, D., Farrand, P., Stuart, G., & Morris, N. (1995). Functional equivalence of verbal and spatial information in serial short-term memory. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *21*(4), 1008.

Jones, D. M., Hughes, R. W., & Macken, W. J. (2006). Perceptual organization masquerading as phonological storage: Further support for a perceptual-gestural view of short-term memory. *Journal of Memory and Language*, *54*(2), 265–281. doi:https://doi.org/10.1016/j.jml.2005.10.006

Jordan, N. C., Glutting, J., & Ramineni, C. (2008). 3 - A Number Sense Assessment Tool for Identifying Children at Risk for Mathematical Difficulties. In A. B. T.-M. D. Dowker (Ed.), *Educational Psychology* (pp. 45–58). San Diego: Academic Press. doi:https://doi.org/10.1016/B978-012373629-1.50005-8

Jordan, N. C., Glutting, J., & Ramineni, C. (2010). The importance of number sense to mathematics achievement in first and third grades. *Learning and Individual Differences*, *20*(2), 82–88. doi:https://doi.org/10.1016/j.lindif.2009.07.004

Jordan, N. C., Glutting, J., Ramineni, C., & Watkins, M. W. (2010). Validating a Number Sense Screening Tool for Use in Kindergarten and First Grade: Prediction of Mathematics Proficiency in Third Grade. *School Psychology Review*, *39*(2), 181–195. doi:10.1080/02796015.2010.12087772

Jordan, N. C., Kaplan, D., Locuniak, M. N., & Ramineni, C. (2007). Predicting First-Grade Math Achievement from Developmental Number Sense Trajectories. *Learning Disabilities Research & Practice*, *22*(1), 36–46. doi:10.1111/j.1540-5826.2007.00229.x

Kabacoff, R. I. (2015). R in action: data analysis and graphics with R, 2nd edition. New York, NY: Manning Publications Co., Shelter Island, New York.

Kadosh, R. C., & Walsh, V. (2007). Dyscalculia. *Current Biology*, *17*(22), R946–R947.

Kail, R. (1991). Developmental change in speed of processing during childhood and adolescence. *Psychological Bulletin*, *109*(3), 490.

Kail, R., & Hall, L. K. (2001). Distinguishing short-term memory from working memory. *Memory & Cognition*, *29*(1), 1–9. doi:10.3758/BF03195735

Kail, R. V, & Ferrer, E. (2007). Processing Speed in Childhood and Adolescence: Longitudinal Models for Examining Developmental Change. *Child Development*, *78*(6), 1760–1770. doi:10.1111/j.1467-8624.2007.01088.x

Kalaman, D. A., & Lefevre, J.-A. (2007). Working memory demands of exact and approximate addition. *European Journal of Cognitive Psychology*, *19*(2), 187–212.

Kamawar, D., LeFevre, J.-A., Bisanz, J., Fast, L., Skwarchuk, S.-L., Smith-Chant, B., & Penner-Wilger, M. (2010). Knowledge of counting principles: How relevant is order irrelevance? *Journal of Experimental Child Psychology*, *105*(1), 138–145. doi:https://doi.org/10.1016/j.jecp.2009.08.004

Kanayet, F. J., Mattarella-Micke, A., Kohler, P. J., Norcia, A. M., McCandliss, B. D., & McClelland, J. L. (2018). Distinct representations of magnitude and spatial position within parietal cortex during number–space mapping. *Journal of Cognitive Neuroscience*, *30*(2), 200–218.

Kane, M. J., Bleckley, M. K., Conway, A. R. A., & Engle, R. W. (2001). A controlled-attention view of working-memory capacity. *Journal of Experimental Psychology: General*, *130*(2), 169.

Kane, M. J., Hambrick, D. Z., Tuholski, S. W., Wilhelm, O., Payne, T. W., & Engle, R. W. (2004). The Generality of Working Memory Capacity: A Latent-Variable Approach to Verbal and Visuospatial Memory Span and Reasoning. *Journal of Experimental Psychology: General*. Kane, Michael J.: Department of Psychology, University of North Carolina at Greensboro, P.O. Box 26170, Greensboro, NC, US, 27402-6170, mjkane@uncg.edu: American Psychological Association. doi:10.1037/0096-3445.133.2.189

Katz, I. R., Bennett, R. E., & Berger, A. E. (2000). Effects of response format on difficulty of SAT-mathematics items: It's not the strategy. *Journal of Educational Measurement*, *37*(1), 39–57.

Keeler, M. L., & Swanson, H. L. (2001). Does strategy knowledge influence working memory in children with mathematical disabilities? *Journal of Learning Disabilities*, *34*(5), 418–434.

Kelly, M. S., & Peverly, S. T. (1992). Identifying bright kindergartners at risk for learning difficulties: Predictive validity of a kindergarten screening tool. *Journal of School Psychology*, *30*(3), 245–258. doi:https://doi.org/10.1016/0022-4405(92)90009-T

Keppel, G., & Underwood, B. J. (1962). Proactive inhibition in short-term retention of single items. *Journal of Verbal Learning and Verbal Behavior*, *1*(3), 153–161.

Klauer, K. C., & Zhao, Z. (2004). Double dissociations in visual and spatial short-term memory. *Journal of Experimental Psychology: General*, *133*(3), 355.

Kline, R. B. (2011). Principles and practice of structural equation modeling (3. Baskı). *New*

York, NY: Guilford.

Knight, P. T. (2001). Complexity and curriculum: A process approach to curriculum-making. *Teaching in Higher Education*, *6*(3), 369–381.

Kobayashi, T., Hiraki, K., & Hasegawa, T. (2005). Auditory–visual intermodal matching of small numerosities in 6-month-old infants. *Developmental Science*, *8*(5), 409–419. doi:10.1111/j.1467-7687.2005.00429.x

Kobayashi, T., Hiraki, K., Mugitani, R., & Hasegawa, T. (2004). Baby arithmetic: One object plus one tone. *Cognition*, *91*(2), B23–B34.

Krajewski, K., & Schneider, W. (2009a). Early development of quantity to number-word linkage as a precursor of mathematical school achievement and mathematical difficulties: Findings from a four-year longitudinal study. *Learning and Instruction*, *19*(6), 513–526. doi:https://doi.org/10.1016/j.learninstruc.2008.10.002

Krajewski, K., & Schneider, W. (2009b). Exploring the impact of phonological awareness, visual–spatial working memory, and preschool quantity–number competencies on mathematics achievement in elementary school: Findings from a 3-year longitudinal study. *Journal of Experimental Child Psychology*, *103*(4), 516–531. doi:https://doi.org/10.1016/j.jecp.2009.03.009

Kroll, N. E., Parks, T., Parkinson, S. R., Bieber, S. L., & Johnson, A. L. (1970). Short-term memory while shadowing: Recall of visually and of aurally presented letters. *Journal of Experimental Psychology*, *85*(2), 220.

Krousel-Wood, M., Muntner, P., Jannu, A., Hyre, A., & Breault, J. (2006). Does Waiver of Written Informed Consent from the Institutional Review Board Affect Response Rate in a Low-Risk Research Study? *Journal of Investigative Medicine*, *54*(4), 174 LP – 179. Retrieved from http://jim.bmj.com/content/54/4/174.abstract

Kühberger, A., Fritz, A., & Scherndl, T. (2014). Publication Bias in Psychology: A Diagnosis Based on the Correlation between Effect Size and Sample Size. *PLOS ONE*, *9*(9), e105825. Retrieved from https://doi.org/10.1371/journal.pone.0105825

Kyllonen, P. C., & Christal, R. E. (1990). Reasoning ability is (little more than) working-memory capacity?! *Intelligence*, *14*(4), 389–433.

Kyttälä, M., Kanerva, K., Munter, I., & Björn, P. M. (2019). Working memory resources in children: stability and relation to subsequent academic skills. *Educational Psychology*, *39*(6), 709–728. doi:10.1080/01443410.2018.1562046

Kyttälä, M., & Lehto, J. E. (2008). Some factors underlying mathematical performance: The role of visuospatial working memory and non-verbal intelligence. *European Journal of Psychology of Education*, *23*(1), 77. doi:10.1007/BF03173141

Lambert, K., & Spinath, B. (2017). Conservation Abilities, Visuospatial Skills, and Numerosity Processing Speed: Association With Math Achievement and Math Difficulties in Elementary School Children. *Journal of Learning Disabilities*, *51*(3), 223–235. doi:10.1177/0022219417690354

Lanfranchi, S., Carretti, B., Spanò, G., & Cornoldi, C. (2009). A specific deficit in visuospatial simultaneous working memory in Down syndrome. *Journal of Intellectual Disability Research*, *53*(5), 474–483.

Lanzi, R. G., & Ramey, S. L. (2013). Maintaining Professionalism, Appropriate Distance, and Consistency in Relationships with Participants in Longitudinal Research: Guidelines for Investigators and Research Staff. *NHSA Dialog*, *16*(4).

Larsen, J. D., & Baddeley, A. (2003). Disruption of verbal STM by irrelevant speech, articulatory suppression, and manual tapping: Do they have a common source? *The Quarterly Journal of Experimental Psychology Section A*, *56*(8), 1249–1268.

Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition*, *105*(2), 395–438.

LeCompte, D. C. (1999). Seven, Plus or Minus Two, is too much to Bear: Three (or Fewer) is the Real Magic Number. *Proceedings of the Human Factors and Ergonomics Society Annual Meeting*, *43*(3), 289–292. doi:10.1177/154193129904300334

Lee, K.-M., & Kang, S.-Y. (2002). Arithmetic operation and working memory: differential suppression in dual tasks. *Cognition*, *83*(3), B63–B68. doi:https://doi.org/10.1016/S0010-0277(02)00010-0

LeFevre, J.-A., Smith-Chant, B. L., Fast, L., Skwarchuk, S.-L., Sargla, E., Arnup, J. S., … Kamawar, D. (2006). What counts as knowing? The development of conceptual and procedural knowledge of counting from kindergarten through Grade 2. *Journal of Experimental Child Psychology*, *93*(4), 285–303. doi:https://doi.org/10.1016/j.jecp.2005.11.002

Lenhard, W., & Lenhard, A. (2016). Calculation of Effect Sizes. Retrieved from https://www.psychometrica.de/effect_size.html.

Levine, T. R., Asada, K. J., & Carpenter, C. (2009). Sample Sizes and Effect Sizes are Negatively Correlated in Meta-Analyses: Evidence and Implications of a Publication Bias Against NonSignificant Findings. *Communication Monographs*, *76*(3), 286–302. doi:10.1080/03637750903074685

Li, Y., & Geary, D. C. (2013). Developmental Gains in Visuospatial Memory Predict Gains in Mathematics Achievement. *PLOS ONE*, *8*(7). doi:10.1371/journal.pone.0070160

Li, Y., & Geary, D. C. (2017). Children’s visuospatial memory predicts mathematics achievement through early adolescence. *PLOS ONE*, *12*(2). doi:10.1371/journal.pone.0172046

Libertus, M. E., Feigenson, L., & Halberda, J. (2011). Preschool acuity of the approximate number system correlates with school math ability. *Developmental Science*, *14*(6), 1292–1300. doi:10.1111/j.1467-7687.2011.01080.x

Lipton, J. S., & Spelke, E. S. (2003). Origins of number sense: Large-number discrimination in human infants. *Psychological Science*, *14*(5), 396–401.

Loehlin, J. C., & Beaujean, A. A. (2016). *Latent variable models: An introduction to factor, path, and structural equation analysis*. Taylor & Francis.

Logie, R. H. (1986). Visuo-spatial processing in working memory. *The Quarterly Journal of Experimental Psychology Section A*, *38*(2), 229–247.

Logie, R. H. (1995). *Visuo-spatial working memory*. Psychology Press.

Logie, R. H. (2001). Working memory: A mental space for design and discovery.

Logie, R. H., Gilhooly, K. J., & Wynn, V. (1994). Counting on working memory in arithmetic problem solving. *Memory & Cognition*, *22*(4), 395–410. doi:10.3758/BF03200866

Luzzatti, C., Vecchi, T., Agazzi, D., Cesa-Bianchi, M., & Vergani, C. (1998). A Neurological Dissociation Between Preserved Visual and Impaired Spatial Processing in Mental Imagery. *Cortex*, *34*(3), 461–469. doi:https://doi.org/10.1016/S0010-9452(08)70768-8

MacPherson, S. E., Sala, S. Della, Logie, R. H., & Wilcock, G. K. (2007). Specific AD Impairment in Concurrent Performance of Two Memory Tasks. *Cortex*, *43*(7), 858–865. doi:https://doi.org/10.1016/S0010-9452(08)70685-3

Maennamaa, M., Kikasb, E., Peets, K., & Palu, A. (2012). Cognitive correlates of math skills in third-grade students. *EDUCATIONAL PSYCHOLOGY*, *32*(1), 21–44. doi:10.1080/01443410.2011.621713

Mahon, A., Glendinning, C., Clarke, K., & Craig, G. (1996). Researching Children: Methods and Ethics. *Children & Society*, *10*(2), 145–154. doi:10.1111/j.1099-0860.1996.tb00464.x

Mammarella, I. C., Borella, E., Pastore, M., & Pazzaglia, F. (2013). The structure of visuospatial memory in adulthood. *Learning and Individual Differences*, *25*, 99–110. doi:https://doi.org/10.1016/j.lindif.2013.01.014

Mammarella, I. C., Caviola, S., Cornoldi, C., & Lucangeli, D. (2013). Mental additions and verbal-domain interference in children with developmental dyscalculia. *Research in Developmental Disabilities*, *34*(9), 2845–2855.

Mammarella, I. C., Caviola, S., Giofrè, D., & Szűcs, D. (2017). The underlying structure of visuospatial working memory in children with mathematical learning disability. *British Journal of Developmental Psychology*, *36*(2). doi:10.1111/bjdp.12202

Mammarella, I. C., Caviola, S., Giofrè, D., & Szűcs, D. (2018). The underlying structure of visuospatial working memory in children with mathematical learning disability. *British Journal of Developmental Psychology*, *36*(2), 220–235.

Mammarella, I. C., Cornoldi, C., Pazzaglia, F., Toso, C., Grimoldi, M., & Vio, C. (2006). Evidence for a double dissociation between spatial-simultaneous and spatial-sequential working memory in visuospatial (nonverbal) learning disabled children. *Brain and Cognition*, *62*(1), 58–67. doi:https://doi.org/10.1016/j.bandc.2006.03.007

Mammarella, I. C., Giofrè, D., & Caviola, S. (2017). Learning geometry: The development of geometrical concepts and the role of cognitive processes. In *Acquisition of Complex Arithmetic Skills and Higher-Order Mathematics Concepts* (pp. 221–246). Elsevier.

Mammarella, I. C., Giofre, D., Ferrara, R., & Cornoldi, C. (2013). Intuitive geometry and visuospatial working memory in children showing symptoms of nonverbal learning disabilities. *CHILD NEUROPSYCHOLOGY*, *19*(3), 235–249. doi:10.1080/09297049.2011.640931

Mammarella, I. C., Hill, F., Devine, A., Caviola, S., & Szucs, D. (2015). Math anxiety and developmental dyscalculia: A study on working memory processes. *JOURNAL OF CLINICAL AND EXPERIMENTAL NEUROPSYCHOLOGY*, *37*(8), 878–887. doi:10.1080/13803395.2015.1066759

Mammarella, I. C., Pazzaglia, F., & Cornoldi, C. (2008). Evidence for different components in children's visuospatial working memory. *British Journal of Developmental Psychology*, *26*(3), 337–355. doi:10.1348/026151007X236061

Martin, R. B., Cirino, P. T., Sharp, C., & Barnes, M. (2014). Number and counting skills in kindergarten as predictors of grade 1 mathematical skills. *Learning and Individual Differences*, *34*(Supplement C), 12–23. doi:https://doi.org/10.1016/j.lindif.2014.05.006

Maybery, M. T., & Do, N. (2003). Relationships between facets of working memory and performance on a curriculum-based mathematics test in children. *Educational and Child Psychology*, *20*(3), 77–92. Retrieved from https://www.researchgate.net/profile/Lorna_Bourke/publication/250928054_The_relationship_between_working_memory_and_early_writing_at_the_word_sentence_and_text_level/links/02e7e51ee5262398b0000000.pdf#page=79

Mazzocco, M. M. M., Feigenson, L., & Halberda, J. (2011). Preschoolers' Precision of the Approximate Number System Predicts Later School Mathematics Performance. *PLOS ONE*, *6*(9), e23749. Retrieved from https://doi.org/10.1371/journal.pone.0023749

McDowell, S., Whyte, J., & D'Esposito, M. (1997). Working memory impairments in traumatic brain injury: evidence from a dual-task paradigm. *Neuropsychologia*, *35*(10), 1341–1353. doi:https://doi.org/10.1016/S0028-3932(97)00082-1

Mcintosh, A., Reys, B. J., & Reys, R. E. (1992). A Proposed Framework for Examining Basic Number Sense. *For the Learning of Mathematics*, *12*(3), 2–44. Retrieved from http://www.jstor.org/stable/40248053

McKenzie, B., Bull, R., & Gray, C. (2003). The effects of phonological and visual-spatial interference on children's arithmetic performance. *Educational and Child Psychology*, *20*, 93–108.

McLean, J F, & Hitch, G. J. (1999). Working memory impairments in children with specific arithmetic learning difficulties. *Journal of Experimental Child Psychology*, *74*(3), 240–260. doi:10.1006/jecp.1999.2516

McLean, Janet F, & Hitch, G. J. (1999). Working Memory Impairments in Children with Specific Arithmetic Learning Difficulties. *Journal of Experimental Child Psychology*, *74*(3), 240–260. doi:https://doi.org/10.1006/jecp.1999.2516

Meyer, M. L., Salimpoor, V. N., Wu, S. S., Geary, D. C., & Menon, V. (2010). Differential contribution of specific working memory components to mathematics achievement in 2nd and 3rd graders. *Learning and Individual Differences*, *20*(2), 101–109. doi:10.1016/j.lindif.2009.08.004

Miller, G A, & Galanter, E. P. (1960). KM Plans and the Structure of Behaviour. Holt Rinehart and Winston.

Miller, George A. (1956). The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychological Review*, *63*(2), 81.

Mix, K. S., Levine, S. C., Cheng, Y.-L., Young, C., Hambrick, D. Z., Ping, R., & Konstantopoulos, S. (2016). Separate but correlated: The latent structure of space and mathematics across development. *Journal of Experimental Psychology: General*. Mix, Kelly S.: Department of Human Development and Quantitative Methods, College of Education, University of Maryland, 3304 Benjamin Building, College Park, MD, US, 20742, kmix@umd.edu: American Psychological Association. doi:10.1037/xge0000182

Monaco, M., Costa, A., Caltagirone, C., & Carlesimo, G. A. (2013). Forward and backward span for verbal and visuo-spatial data: standardization and normative data from an Italian adult population. *Neurological Sciences*, *34*(5), 749–754. doi:10.1007/s10072-012-1130-x

Morey, C. C., Mareva, S., Lelonkiewicz, J. R., & Chevalier, N. (2018). Gaze-based rehearsal in children under 7: a developmental investigation of eye movements during a serial spatial memory task. *Developmental Science*, *21*(3), e12559. doi:10.1111/desc.12559

Morra, S., Bisagno, E., Caviola, S., Delfante, C., & Mammarella, I. C. (2019). Working memory capacity and the development of quantitative central conceptual structures. *Cognition and Instruction*, *37*(4), 483–511.

Morris, R. G. (1994). Working memory in Alzheimer-type dementia. *Neuropsychology*, *8*(4), 544.

Morrison, A. B., & Chein, J. M. (2011). Does working memory training work? the promise and challenges of enhancing cognition by training working memory. *Psychonomic Bulletin and Review*. doi:10.3758/s13423-010-0034-0

Morrow, V., & Richards, M. (1996). The Ethics of Social Research with Children: An Overview1. *Children & Society*, *10*(2), 90–105. doi:10.1111/j.1099-0860.1996.tb00461.x

Nath, S., & Szücs, D. (2014). Construction play and cognitive skills associated with the development of mathematical abilities in 7-year-old children. *Learning and Instruction*, *32*(Supplement C), 73–80. doi:https://doi.org/10.1016/j.learninstruc.2014.01.006

Nosworthy, N., Bugden, S., Archibald, L., Evans, B., & Ansari, D. (2013). A Two-Minute Paper-and-Pencil Test of Symbolic and Nonsymbolic Numerical Magnitude Processing Explains Variability in Primary School Children's Arithmetic Competence. *PLOS ONE*, *8*(7), e67918. Retrieved from https://doi.org/10.1371/journal.pone.0067918

O'Neil Jr., H. F., & Brown, R. S. (1998). Differential Effects of Question Formats in Math Assessment on Metacognition and Affect. *Applied Measurement in Education*, *11*(4), 331–351. doi:10.1207/s15324818ame1104_3

Odic, D., Lisboa, J. V., Eisinger, R., Olivera, M. G., Maiche, A., & Halberda, J. (2016). Approximate number and approximate time discrimination each correlate with school math abilities in young children. *Acta Psychologica*, *163*, 17–26. doi:https://doi.org/10.1016/j.actpsy.2015.10.010

Olkun, S., Altun, A., Gocer Sahin, S., & Kaya, G. (2016). Psychometric Properties of a Screening Tool for Elementary School Student's Math Learning Disorder Risk. *Online Submission*, *15*(12), 48–66.

Onwuegbuzie, A. J., & Seaman, M. A. (1995). The Effect of Time Constraints and Statistics Test Anxiety on Test Performance in a Statistics Course. *The Journal of Experimental Education*, *63*(2), 115–124. doi:10.1080/00220973.1995.9943816

Ottem, E. J., Lian, A., & Karlsen, P. J. (2007). Reasons for the growth of traditional memory span across age. *European Journal of Cognitive Psychology*, *19*(2), 233–270.

Pascual-Leone, J. (1970). A mathematical model for the transition rule in Piaget's developmental stages. *Acta Psychologica*, *32*, 301–345.

Passolunghi, M C, & Siegel, L. S. (2001). Short term memory, working memory, and inhibitory control in children with specific arithmetic learning disabilities. *Journal of Experimental Child Psychology*, *80*(1), 44–57.

Passolunghi, M Chiara, Mammarella, I. C., & Altoè, G. (2008). Cognitive Abilities as Precursors of the Early Acquisition of Mathematical Skills During First Through Second Grades. *Developmental Neuropsychology*, *33*(3), 229–250. doi:10.1080/87565640801982320

Passolunghi, Maria Chiara, & Cornoldi, C. (2008). Working Memory Failures in Children with Arithmetical Difficulties. *Child Neuropsychology*, *14*(5), 387–400. doi:10.1080/09297040701566662

Passolunghi, Maria Chiara, & Costa, H. M. (2019). Working Memory and Mathematical Learning BT - International Handbook of Mathematical Learning Difficulties: From the Laboratory to the Classroom. In A. Fritz, V. G. Haase, & P. Räsänen (Eds.), (pp. 407–421). Cham: Springer International Publishing. doi:10.1007/978-3-319-97148-3_25

Passolunghi, Maria Chiara, & Mammarella, I. C. (2010). Spatial and visual working memory ability in children with difficulties in arithmetic word problem solving. *EUROPEAN JOURNAL OF COGNITIVE PSYCHOLOGY*, *22*(6), 944–963. doi:10.1080/09541440903091127

Passolunghi, Maria Chiara, & Mammarella, I. C. (2011). Selective Spatial Working Memory Impairment in a Group of Children With Mathematics Learning Disabilities and Poor Problem-Solving Skills. *Journal of Learning Disabilities*, *45*(4), 341–350. doi:10.1177/0022219411400746

Passolunghi, Maria Chiara, & Mammarella, I. C. (2012). Selective Spatial Working Memory Impairment in a Group of Children With Mathematics Learning Disabilities and Poor Problem-Solving Skills. *JOURNAL OF LEARNING DISABILITIES*, *45*(4), 341–350. doi:10.1177/0022219411400746

Pazzaglia, F. (1999). The role of distinct components of visuo-spatial working memory in the processing of texts. *Memory*, *7*(1), 19–41.

Pedhazur, E. J. (1997). Multiple regression in behavioral research: Explanation and prediction . Thompson Learning. *Inc: New York, NY.*

Pelegrina, S., Capodieci, A., Carretti, B., & Cornoldi, C. (2015). Magnitude representation and working memory updating in children with arithmetic and reading comprehension disabilities. *Journal of Learning Disabilities*, *48*(6), 658–668.

Peng, P., & Fuchs, D. (2014). A Meta-Analysis of Working Memory Deficits in Children With Learning Difficulties: Is There a Difference Between Verbal Domain and Numerical Domain? *Journal of Learning Disabilities*, *49*(1), 3–20. doi:10.1177/0022219414521667

Peng, P., & Fuchs, D. (2016). A meta-analysis of working memory deficits in children with learning difficulties: Is there a difference between verbal domain and numerical domain? *Journal of Learning Disabilities*, *49*(1), 3–20.

Peng, P., Namkung, J., Barnes, M., & Sun, C. (2016). A Meta-Analysis of Mathematics and Working Memory: Moderating Effects of Working Memory Domain, Type of Mathematics Skill, and Sample Characteristics. *JOURNAL OF EDUCATIONAL*

PSYCHOLOGY, *108*(4), 455–473. doi:10.1037/edu0000079

Peters, G.-J. Y. (2014). The alpha and the omega of scale reliability and validity: Why and how to abandon Chronbach's alpha and the route towards more comprehensive assessment of scale quality. *European Health Psychologist*, *16*(2), 56–69. doi:10.31234/osf.io/h47fv

Peterson, K. M. H., & Shinn, M. R. (2002). Severe discrepancy models: Which best explains school identification practices for learning disabilities? *School Psychology Review*, *31*(4), 459–476.

Phillips, W. A. (1974). On the distinction between sensory storage and short-term visual memory. *Perception & Psychophysics*, *16*(2), 283–290.

Phillips, W. A., & Baddeley, A. D. (1971). Reaction time and short-term visual memory. *Psychonomic Science*, *22*(2), 73–74. doi:10.3758/BF03332500

Pickering, S. J., Gathercole, S. E., & Peaker, S. M. (1998). Verbal and visuospatial short-term memory in children: Evidence for common and distinct mechanisms. *Memory & Cognition*, *26*(6), 1117–1130.

Pina, V., Fuentes, L. J., Castillo, A., & Diamantopoulou, S. (2014). Disentangling the effects of working memory, language, parental education, and non-verbal intelligence on children's mathematical abilities. *Frontiers in Psychology*, *5*, 415. doi:10.3389/fpsyg.2014.00415

Posner, M. I., & Keele, S. W. (1967). Decay of visual information from a single letter. *Science*, *158*(3797), 137–139.

Powell, M. A., Fitzgerald, R. M., Taylor, N., & Graham, A. (2012). International literature review: Ethical issues in undertaking research with children and young people.

Primi, R., Ferrão, M. E., & Almeida, L. S. (2010). Fluid intelligence as a predictor of learning: A longitudinal multilevel approach applied to math. *Learning and Individual Differences*, *20*(5), 446–451. doi:https://doi.org/10.1016/j.lindif.2010.05.001

Quinn, J. G., & McConnell, J. (2006). The interval for interference in conscious visual imagery. *Memory*, *14*(2), 241–252. doi:10.1080/09658210500210019

R Core Team. (2018). R: A language and environment for statistical computing. Vienna, Austria, Austria: R Foundation for Statistical Computing. Retrieved from http://www.r-project.org/

Raghubar, K. P., Barnes, M. A., & Hecht, S. A. (2010). Working memory and mathematics: A review of developmental, individual difference, and cognitive approaches. *Learning and Individual Differences*, *20*(2), 110–122. doi:https://doi.org/10.1016/j.lindif.2009.10.005

Rasmussen, C., & Bisanz, J. (2005). Representation and working memory in early arithmetic. *Journal of Experimental Child Psychology*, *91*(2), 137–157. doi:https://doi.org/10.1016/j.jecp.2005.01.004

Regard, M., Strauss, E., & Knapp, P. (1982). Children's Production on Verbal and Non-Verbal Fluency Tasks. *Perceptual and Motor Skills*, *55*(3), 839–844. doi:10.2466/pms.1982.55.3.839

RepovŠ, G., & Baddeley, A. (2006). The multi-component model of working memory: Explorations in experimental cognitive psychology. *Neuroscience*, *139*(1), 5–21. doi:https://doi.org/10.1016/j.neuroscience.2005.12.061

Resnick, L. B. (1984). Toward a cognitive theory of instruction. In *Learning and motivation in the classroom* (pp. 5–38). Routledge.

Reuhkala, M. (2001). Mathematical Skills in Ninth-graders: Relationship with visuo-spatial abilities and working memory. *Educational Psychology*, *21*(4), 387–399. doi:10.1080/01443410120090786

Revelle, W. R. (2017). psych: Procedures for personality and psychological research.

Riggs, K. J., McTaggart, J., Simpson, A., & Freeman, R. P. J. (2006). Changes in the capacity of visual working memory in 5-to 10-year-olds. *Journal of Experimental Child Psychology*, *95*(1), 18–26.

Robertson, I. H., Manly, T., Andrade, J., Baddeley, B. T., & Yiend, J. (1997). Oops!': performance correlates of everyday attentional failures in traumatic brain injured and normal subjects. *Neuropsychologia*, *35*(6), 747–758.

Rohde, T. E., & Thompson, L. A. (2007). Predicting academic achievement with cognitive ability. *Intelligence*, *35*(1), 83–92. doi:https://doi.org/10.1016/j.intell.2006.05.004

Rohrkemper, M. (1986). The Functions of Inner Speech in Elementary School Students' Problem-Solving Behavior. *American Educational Research Journal*, *23*(2), 303–313. doi:10.3102/00028312023002303

Rönnberg, J. (2004). Cognitive and neuroscience perspectives on speech and sign processing: Evidence from persons with deafness, hearing impairment, and normal hearing. *International Journal of Psychology*, *39*(5–6), 222.

Rönnberg, J., Rudner, M., & Ingvar, M. (2004). Neural correlates of working memory for sign language. *Cognitive Brain Research*, *20*(2), 165–182.

Ropeter, A., & Pauen, S. (2013). Relating 7-Month-Olds Visuo-Spatial Working Memory to Other Basic Mental Skills Assessed With Two Different Versions of the Habituation–Dishabituation Paradigm. *Infancy*, *18*(4), 578–603. doi:10.1111/j.1532-7078.2012.00133.x

Rosseel, Y. (2010). Mplus estimators: MLM and MLR. In *First Mplus User meeting–October 27th*.

Rosseel, Y. (2012). lavaan: An R Package for Structural Equation Modeling. *Journal of Statistical Software; Vol 1, Issue 2 (2012)* . Retrieved from https://www.jstatsoft.org/v048/i02

Rudkin, S. J., Pearson, D. G., & Logie, R. H. (2007). Executive processes in visual and spatial working memory tasks. *The Quarterly Journal of Experimental Psychology*, *60*(1), 79–100. doi:10.1080/17470210600587976

Sala, G., & Gobet, F. (2017). Working memory training in typically developing children: A meta-analysis of the available evidence. *Developmental Psychology*. Sala, Giovanni: Department of Psychological Sciences, Bedford Street South, University of Liverpool, Liverpool, United Kingdom, L69 7ZA, giovanni.sala@liv.ac.uk: American Psychological Association. doi:10.1037/dev0000265

Salamé, P., & Baddeley, A. (1986). Phonological factors in STM: Similarity and the unattended speech effect. *Bulletin of the Psychonomic Society*, *24*(4), 263–265. doi:10.3758/BF03330135

Salvatore, S., & Valsiner, J. (2010). Between the General and the Unique: Overcoming the Nomothetic versus Idiographic Opposition. *Theory & Psychology*, *20*(6), 817–833. doi:10.1177/0959354310381156

Sarnecka, B. W., & Gelman, S. A. (2004). Six does not just mean a lot: preschoolers see number words as specific. *Cognition*, *92*(3), 329–352. doi:10.1016/j.cognition.2003.10.001

Sarnecka, B., & Wright, C. (2013). The Idea of an Exact Number: Children's Understanding of Cardinality and Equinumerosity. *Cognitive Science*, *37*(8), 1493–1506. doi:10.1111/cogs.12043

Sasanguie, D., Göbel, S. M., Moll, K., Smets, K., & Reynvoet, B. (2013). Approximate number sense, symbolic number processing, or number–space mappings: What underlies mathematics achievement? *Journal of Experimental Child Psychology*, *114*(3), 418–431. doi:https://doi.org/10.1016/j.jecp.2012.10.012

Saults, J. S., & Cowan, N. (2007). A central capacity limit to the simultaneous storage of visual and auditory arrays in working memory. *Journal of Experimental Psychology: General*, *136*(4), 663.

Saywitz, K. J., & Nathanson, R. (1993). Children's testimony and their perceptions of stress in and out of the courtroom. *Child Abuse & Neglect*, *17*(5), 613–622. doi:https://doi.org/10.1016/0145-2134(93)90083-H

Schmidt, W., Houang, R., & Cogan, L. (2002). A coherent curriculum. *American Education*.

Schneider, M., Grabner, R. H., & Paetsch, J. (2009). Mental number line, number line estimation, and mathematical achievement: Their interrelations in grades 5 and 6. *Journal of Educational Psychology*. Schneider, Michael: Institute of Behavioral Sciences, ETH Zurich, Universitatsstrasse 6, Zurich, Switzerland, 8092, schneider@ifv.gess.ethz.ch: American Psychological Association. doi:10.1037/a0013840

Schneider, W. (2008). The Development of Metacognitive Knowledge in Children and Adolescents: Major Trends and Implications for Education. *Mind, Brain, and Education*, *2*(3), 114–121. doi:10.1111/j.1751-228X.2008.00041.x

Schwarzer, G., Carpenter, J. R., & Rücker, G. (2015). Meta-analysis with R (Use-R!). *Switzerland: Springer International Publishing*.

Seethaler, P. M., & Fuchs, L. S. (2010). The Predictive Utility of Kindergarten Screening for Math Difficulty. *Exceptional Children*, *77*(1), 37–59. doi:10.1177/001440291007700102

Shah, P., & Miyake, A. (1996). The separability of working memory resources for spatial thinking and language processing: An individual differences approach. *Journal of Experimental Psychology: General*, *125*(1), 4.

Shiffrin, R. M., & Atkinson, R. C. (1969). Storage and retrieval processes in long-term memory. *Psychological Review*, *76*(2), 179.

Siegel, L. S., & Ryan, E. B. (1989). The Development of Working Memory in Normally Achieving and Subtypes of Learning Disabled Children. *Child Development*, *60*(4), 973–980. doi:10.2307/1131037

Siegler, R. S., & Booth, J. R. (2004). Development of Numerical Estimation in Young Children. *Child Development*, *75*(2), 428–444. doi:10.1111/j.1467-8624.2004.00684.x

Simmons, F. R., Willis, C., & Adams, A.-M. (2012). Different components of working memory have different relationships with different mathematical skills. *Journal of Experimental Child Psychology*, *111*(2), 139–155. doi:https://doi.org/10.1016/j.jecp.2011.08.011

Smith, E. E., Jonides, J., Marshuetz, C., & Koeppe, R. A. (1998). Components of verbal working memory: evidence from neuroimaging. *Proceedings of the National Academy of Sciences*, *95*(3), 876–882.

Snider, V. E. (2004). A comparison of spiral versus strand curriculum. *Journal of Direct Instruction*, *4*(1), 29–39.

Soltanlou, M., Pixner, S., & Nuerk, H.-C. (2015). Contribution of working memory in multiplication fact network in children may shift from verbal to visuo-spatial: a longitudinal investigation. *FRONTIERS IN PSYCHOLOGY*, *6*, 1062. doi:10.3389/fpsyg.2015.01062

Speciale, G., Ellis, N. C., & Bywater, T. (2004). Phonological sequence learning and short-term store capacity determine second language vocabulary acquisition. *Applied Psycholinguistics*, *25*(2), 293–321.

St Clair-Thompson, H. L., & Gathercole, S. E. (2006). Executive functions and achievements in school: Shifting, updating, inhibition, and working memory. *Quarterly Journal of Experimental Psychology*, *59*(4), 745–759.

Starkey, P., & Cooper, R. G. (1980). Perception of numbers by human infants. *Science*, *210*(4473), 1033–1035.

Starkey, P., Spelke, E. S., & Gelman, R. (1990). Numerical abstraction by human infants. *Cognition*, *36*(2), 97–127. doi:https://doi.org/10.1016/0010-0277(90)90001-Z

Starr, A., Libertus, M. E., & Brannon, E. M. (2013). Number sense in infancy predicts mathematical abilities in childhood. *Proceedings of the National Academy of Sciences*, *110*(45), 18116–18120.

Steel, S., & Funnell, E. (2001). Learning multiplication facts: A study of children taught by discovery methods in England. *Journal of Experimental Child Psychology*, *79*(1), 37–55.

Stock, P., Desoete, A., & Roeyers, H. (2009). Mastery of the counting principles in toddlers: A crucial step in the development of budding arithmetic abilities? *Learning and Individual Differences*, *19*(4), 419–422.

Strauss, M. S., & Curtis, L. E. (1981). Infant perception of numerosity. *Child Development*, 1146–1152.

Swanson, H. L. (1993). Working memory in learning disability subgroups. *Journal of Experimental Child Psychology*, *56*(1), 87–114.

Swanson, H. L., & Beebe-Frankenberger, M. (2004). The Relationship Between Working Memory and Mathematical Problem Solving in Children at Risk and Not at Risk for Serious Math Difficulties. *Journal of Educational Psychology*, *96*(3), 471–491. doi:10.1037/0022-0663.96.3.471

Swanson, H. L., & Jerman, O. (2006). Math disabilities: A selective meta-analysis of the literature. *Review of Educational Research*, *76*(2), 249–274.

Swanson, H. L., Jerman, O., & Zheng, X. (2008). Growth in working memory and mathematical problem solving in children at risk and not at risk for serious math difficulties. *Journal of Educational Psychology*, *100*(2), 343.

Swanson, H. L., & Sachse-Lee, C. (2001a). A subgroup analysis of working memory in children with reading disabilities: Domain-general or domain-specific deficiency? *Journal of Learning Disabilities*, *34*(3), 249–263.

Swanson, H. L., & Sachse-Lee, C. (2001b). Mathematical Problem Solving and Working Memory in Children with Learning Disabilities: Both Executive and Phonological Processes Are Important. *Journal of Experimental Child Psychology*, *79*(3), 294–321. doi:10.1006/JECP.2000.2587

Swanson, L., & Kim, K. (2007). Working memory, short-term memory, and naming speed as predictors of children's mathematical performance. *Intelligence*, *35*(2), 151–168. doi:https://doi.org/10.1016/j.intell.2006.07.001

Sweller, J. (1994). Cognitive load theory, learning difficulty, and instructional design. *Learning and Instruction*, *4*(4), 295–312. doi:https://doi.org/10.1016/0959-4752(94)90003-5

Szucs, D., Devine, A., Soltesz, F., Nobes, A., & Gabriel, F. (2013). Developmental dyscalculia is related to visuo-spatial memory and inhibition impairment. *Cortex; a Journal Devoted to the Study of the Nervous System and Behavior*, *49*(10), 2674–2688. doi:10.1016/j.cortex.2013.06.007

Szűcs, D. (2016). Chapter 11 - Subtypes and comorbidity in mathematical learning disabilities: Multidimensional study of verbal and visual memory processes is key to understanding. In M. Cappelletti & W. B. T.-P. in B. R. Fias (Eds.), *The Mathematical Brain Across the Lifespan* (Vol. 227, pp. 277–304). Elsevier. doi:https://doi.org/10.1016/bs.pbr.2016.04.027

Szűcs, Dénes, & Csépe, V. (2004a). *Access to numerical information is dependent on the modality of stimulus presentation in mental addition: A combined ERP and behavioral study*. *Brain research. Cognitive brain research* (Vol. 19). doi:10.1016/j.cogbrainres.2003.11.002

Szűcs, Dénes, & Csépe, V. (2004b). Similarities and differences in the coding of numerical and alphabetical order using acoustic stimulation as revealed by event-related potentials in humans. *Neuroscience Letters*, *360*(1), 65–68. doi:https://doi.org/10.1016/j.neulet.2004.02.038

Tan, L. H., Liu, H.-L., Perfetti, C. A., Spinks, J. A., Fox, P. T., & Gao, J.-H. (2001). The Neural System Underlying Chinese Logograph Reading. *NeuroImage*, *13*(5), 836–846. doi:https://doi.org/10.1006/nimg.2001.0749

Thompson, I. (2010). The principal counting principles. *National Centre for Excellence in the*

Teaching of Mathematics, Early Years Magazine, *7*.

Thompson, P. W. (1996). Imagery and the development of mathematical reasoning. In *Theories of learning mathematics*.

Towse, J N, & Hitch, G. J. (1995). Is there a relationship between task demand and storage space in tests of working memory capacity? *The Quarterly Journal of Experimental Psychology*, *48*(1), 108–124.

Towse, John N, Hitch, G. J., & Hutton, U. (1998). A Reevaluation of Working Memory Capacity in Children. *Journal of Memory and Language*, *39*(2), 195–217. doi:https://doi.org/10.1006/jmla.1998.2574

Towse, John N, Hitch, G. J., & Skeates, S. (1999). Developmental sensitivity to temporal grouping effects in short-term memory. *International Journal of Behavioral Development*, *23*(2), 391–411.

Tremblay, S., Saint-Aubin, J., & Jalbert, A. (2006). Rehearsal in serial memory for visual-spatial information: Evidence from eye movements. *Psychonomic Bulletin & Review*, *13*(3), 452–457.

Tresch, M. C., Sinnamon, H. M., & Seamon, J. G. (1993). Double dissociation of spatial and object visual memory: Evidence from selective interference in intact human subjects. *Neuropsychologia*, *31*(3), 211–219. doi:https://doi.org/10.1016/0028-3932(93)90085-E

Tulving, E., & Patkau, J. E. (1962). Concurrent effects of contextual constraint and word frequency on immediate recall and learning of verbal material. *Canadian Journal of Psychology/Revue Canadienne de Psychologie*, *16*(2), 83.

Unsworth, N., & Engle, R. W. (2006). Simple and complex memory spans and their relation to fluid abilities: Evidence from list-length effects. *Journal of Memory and Language*, *54*(1), 68–80.

Unsworth, N., & Engle, R. W. (2007). On the division of short-term and working memory: an examination of simple and complex span and their relation to higher order abilities. *Psychological Bulletin*, *133*(6), 1038.

Van de Weijer-Bergsma, E., Kroesbergen, E. H., & Van Luit, J. E. H. (2015). Verbal and visual-spatial working memory and mathematical ability in different domains throughout primary school. *Memory & Cognition*, *43*(3), 367–378. doi:10.3758/s13421-014-0480-4

van der Ven, S. H. G., van der Maas, H. L. J., Straatemeier, M., & Jansen, B. R. J. (2013). Visuospatial working memory and mathematical ability at different ages throughout primary school. *Learning and Individual Differences*, *27*(Supplement C), 182–192. doi:https://doi.org/10.1016/j.lindif.2013.09.003

Vanbinst, K., Ceulemans, E., Peters, L., Ghesquière, P., & De Smedt, B. (2018). Developmental trajectories of children's symbolic numerical magnitude processing skills and associated cognitive competencies. *Journal of Experimental Child Psychology*, *166*(Supplement C), 232–250. doi:https://doi.org/10.1016/j.jecp.2017.08.008

Vanbinst, K., Ghesquière, P., & De Smedt, B. (2015). Does numerical processing uniquely predict first graders' future development of single-digit arithmetic? *Learning and Individual Differences*, *37*, 153–160. doi:https://doi.org/10.1016/j.lindif.2014.12.004

Vandenbroucke, L., Verschueren, K., Desoete, A., Aunio, P., Ghesquière, P., & Baeyens, D. (2018). Crossing the bridge to elementary school: The development of children's working memory components in relation to teacher-student relationships and academic achievement. *Early Childhood Research Quarterly*, *42*(Supplement C), 1–10. doi:https://doi.org/10.1016/j.ecresq.2017.08.004

Vecchi, T., & Cornoldi, C. (1999). Passive storage and active manipulation in visuo-spatial working memory: Further evidence from the study of age differences. *European Journal of Cognitive Psychology*, *11*(3), 391–406.

Vecchi, T., Richardson, J., & Cavallini, E. (2005). Passive storage versus active processing in working memory: Evidence from age-related variations in performance. *European Journal of Cognitive Psychology*, *17*(4), 521–539.

Ventre-Dominey, J., Bailly, A., Lavenne, F., LeBars, D., Mollion, H., Costes, N., & Dominey, P. F. (2005). Double dissociation in neural correlates of visual working memory: A PET study. *Cognitive Brain Research*, *25*(3), 747–759. doi:https://doi.org/10.1016/j.cogbrainres.2005.09.004

Vergauwe, E., Barrouillet, P., & Camos, V. (2009). Visual and spatial working memory are not that dissociated after all: A time-based resource-sharing account. *Journal of Experimental Psychology: Learning, Memory, and Cognition*. Vergauwe, Evie: Department of Psychology, University of Geneva, 40 Boulevard du Pont d'Arve, Geneva, Switzerland, 1205, Evie.Vergauwe@unige.ch: American Psychological Association. doi:10.1037/a0015859

Vergauwe, E., Barrouillet, P., & Camos, V. (2010). Do Mental Processes Share a Domain-General Resource? *Psychological Science*, *21*(3), 384–390. doi:10.1177/0956797610361340

Vicari, S., Bellucci, S., & Carlesimo, G. A. (2003). Visual and spatial working memory dissociation: evidence from Williams syndrome. *Developmental Medicine & Child Neurology*, *45*(4), 269–273. doi:DOI: 10.1017/S0012162203000513

Viechtbauer, W. (2010). Conducting meta-analyses in R with the metafor package. *Journal of Statistical Software*, *36*(3), 1–48.

Vogel, E. K., Woodman, G. F., & Luck, S. J. (2001). Storage of features, conjunctions, and objects in visual working memory. *Journal of Experimental Psychology: Human Perception and Performance*, *27*(1), 92.

von Hippel, P. T. (2015). The heterogeneity statistic I(2) can be biased in small meta-analyses. *BMC Medical Research Methodology*, *15*, 35. doi:10.1186/s12874-015-0024-z

Wagner, S. H., & Walters, J. (1982). A longitudinal analysis of early number concepts: From numbers to number. *Action and Thought: From Sensorimotor Schemes to Symbolic Operations*, 137–161.

Wansard, M., Bartolomeo, P., Bastin, C., Segovia, F., Gillet, S., Duret, C., & Meulemans, T. (2015). Support for distinct subcomponents of spatial working memory: A double dissociation between spatial–simultaneous and spatial–sequential performance in unilateral neglect. *Cognitive Neuropsychology*, *32*(1), 14–28. doi:10.1080/02643294.2014.995075

Warfield, J. N. (1988). THE MAGICAL NUMBER THREE-PLUS OR MINUS ZERO.

Cybernetics and Systems, *19*(4), 339–358. doi:10.1080/01969728808902173

Watson, I. (1980). Investigating errors of beginning mathematicians. *Educational Studies in Mathematics*, *11*(3), 319–329.

Wechsler, D. (2017). Wechsler Individual Achievement Test - Third UK Edition (WIAT-III UK) | Pearson Assessment. Retrieved from https://www.pearsonclinical.co.uk/Psychology/ChildCognitionNeuropsychologyandLanguage/ChildAchievementMeasures/wiat-iii/wechsler-individual-achievement-test-third-uk-edition-wiat-iii-uk.aspx

Wellington, J. J., Bathmaker, A.-M., Hunt, C., McCulloch, G., & Sikes, P. (2005). *Succeeding with your Doctorate*. Sage.

Wiklund-Hörnqvist, C., Jonsson, B., Korhonen, J., Eklöf, H., & Nyroos, M. (2016). Untangling the Contribution of the Subcomponents of Working Memory to Mathematical Proficiency as Measured by the National Tests: A Study among Swedish Third Graders. *Frontiers in Psychology*, *7*, 1062. doi:10.3389/fpsyg.2016.01062

Wilson, D. B. (n.d.). Practical meta-analysis effect size calculator (online calculator). Retrieved from https://www.campbellcollaboration.org/effect-size-calculato.html

Wilson, K. M., & Swanson, H. L. (2001). Are Mathematics Disabilities Due to a Domain-General or a Domain-Specific Working Memory Deficit? *Journal of Learning Disabilities*, *34*(3), 237–248. doi:10.1177/002221940103400304

Wong, B., & Szücs, D. (2013). Single-digit Arabic numbers do not automatically activate magnitude representations in adults or in children: Evidence from the symbolic same–different task. *Acta Psychologica*, *144*(3), 488–498.

Wright, H., Dorsett, R., Anders, J., Buzzeo, J., Runge, J., & Sanders, M. (2019). IMPROVING WORKING MEMORY.

Wynn, K. (1990). Children's understanding of counting. *Cognition*, *36*(2), 155–193. doi:https://doi.org/10.1016/0010-0277(90)90003-3

Wynn, K. (1992). Children's acquisition of the number words and the counting system. *Cognitive Psychology*, *24*(2), 220–251. doi:https://doi.org/10.1016/0010-0285(92)90008-P

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition*, *74*(1), B1–B11. doi:https://doi.org/10.1016/S0010-0277(99)00066-9

Zheng, X., Swanson, H. L., & Marcoulides, G. A. (2011). Working memory components as predictors of children's mathematical word problem solving. *Journal of Experimental Child Psychology*, *110*(4), 481–498. doi:10.1016/J.JECP.2011.06.001

Zimmer, H. D. (2008). Visual and spatial working memory: From boxes to networks. *Neuroscience & Biobehavioral Reviews*, *32*(8), 1373–1395. doi:https://doi.org/10.1016/j.neubiorev.2008.05.016

Zinbarg, R. E., Revelle, W., Yovel, I., & Li, W. (2005). Cronbach's α, Revelle's β, and McDonald's ωh: Their relationship with each other and two alternative conceptualizations of reliability. *Psychometrika*, *70*(1), 123–133.

19

Appendices

Appendix A: Ethical approval for study 1

Durham University
School of Education

Shaped by the past, creating the future

1 December 2017

Katie Allen
Katie.allen@durham.ac.uk

Dear Katie

TITLE Understanding the components of visuospatial working memory and their relation to maths attainment

Reference : 2909

I am pleased to inform you that your ethics application for the above research project has been approved by the School of Education Ethics Committee.

May we take this opportunity to wish you good luck with your research.

Yours sincerely,

Dr Nadin Beckmann
School of Education Ethics Committee Chair

Leazes Road
Durham, DH1 1TA
Telephone +44 (0)191 334 2000 Fax +44 (0)191 334 8311
www.durham.ac.uk/education

Appendix B: Ethical approval for study 2

Durham University

School of Education

Shaped by the past, creating the future

31/10/2018

Katie Allen
katie.allen@durham.ac.uk

Dear Katie

Understanding the components of working memory and their relation to maths attainment across the primary school years

Reference: 3191

I am pleased to inform you that your ethics application for the above research project has been approved by the School of Education Ethics Committee.

May we take this opportunity to wish you good luck with your research.

Yours sincerely,

Dr Nadin Beckmann
School of Education Ethics Committee Chair

Leazes Road
Durham, DH1 1TA
Telephone +44 (0)191 334 2000 Fax +44 (0)191 334 8311
www.durham.ac.uk/education

Appendix C: Ethical approval for study 3

Dear Katie,

The following project has received ethical approval:

Project Title: *Feasibility study for the generation of a screening measure for future mathematic attainment ;*
Start Date: *10 June 2019*;
End Date: *19 July 2019;*
Reference: *EDU-2019-04-19T10:26:42-pdgf74*
Date of ethical approval: *24 May 2019*.

Please be aware that if you make any significant changes to the design, duration or delivery of your project, you should contact ed.ethics@durham.ac.uk for advice, as further consideration and approval may then be required.

If you have any queries regarding this approval or need anything further, please contact ed.ethics@durham.ac.uk

Appendix D: Ethical approval for study 4

Dear Katie,

The following project has received ethical approval:

Project Title: *Predicting mathematics attainment from previous working memory performance: 2 year mathematics follow up ;*
Start Date: *01 September 2019*;
End Date: *01 October 2019;*
Reference: *EDU-2019-07-17T08:41:05-pdgf74*
Date of ethical approval: *19 August 2019*.

Please be aware that if you make any significant changes to the design, duration or delivery of your project, you should contact ed.ethics@durham.ac.uk for advice, as further consideration and approval may then be required.

If you have any queries regarding this approval or need anything further, please contact ed.ethics@durham.ac.uk

Appendix E: Permission letter for study 1

Dear parent/ guardian,

I am a postgraduate student at Durham University, studying for my PhD. For my project, I am investigating the relationship between working memory and the different elements of maths. I am hoping to test the children in Year 3 at [SCHOOL NAME] and request your permission for your child's participation in the study. The study complies with the ethical guidelines of the British Psychological Society for psychological research and the British Education Research Association (BERA).

The aim of the research is to explore ways in which working memory relates to maths performance in children over development. Your child will be asked to complete subtests from a working memory battery which will assess their verbal and visual working memory. The children will also be asked to complete an age appropriate assessment relating to the different elements of maths. These tasks will be completed over two sessions on different days. The complete test will take approximately 45 minutes, with the opportunity for them to withdraw at any point, should they feel they do not wish to continue, without giving reason and without any repercussions. Particular attention will be paid to any visible distressed shown by the children, at which point they will be removed from the study environment and their data destroyed. All information will be held confidentially in accordance with the Data Protection Act and will be anonymously coded. Any parents/ guardians/ children wishing for data to be removed from the study may contact one of the researchers and their data will be destroyed.

If you **DO NOT** give permission for your child to participate, please return the following consent form to the school before [DATE]. If the consent form is not returned before this date, I will assume your permission has been granted for your child's participation.

For further information, or to ask any questions, contact details are provided below.

Yours sincerely,

Katie Allen

katie.allen@durham.ac.uk
s.e.higgins@durham.ac.uk (Professor Steve Higgins, Supervisor)

I **DO NOT** give permission for (name of child) ____________________ to participate in the above study

Name of parent/ guardian: ______________________

Parent/ guardian signature: __________________

Date: _________________

Appendix F: Permission letter for study 2

Dear parent/ guardian,

I am a postgraduate student at Durham University, studying for my PhD. For my project, I am investigating the relationship between working memory and the different elements of maths. I am hoping to test the children in Years 2, 3, 4, and 5 at [SCHOOL NAME] and request your permission for your child's participation in the study. The study complies with the ethical guidelines of the British Psychological Society (BPS) for psychological research and the British Education Research Association (BERA).

The aim of the research is to explore ways in which working memory relates to maths performance in children over development. Your child will be asked to complete working memory tasks which will assess their verbal and visual working memory. The children will also be asked to complete an age appropriate assessments relating to the different elements of maths. These tasks will be completed over two sessions on different days. The complete test will take approximately 45 minutes, with the opportunity for them to withdraw at any point, should they feel they do not wish to continue, without giving reason and without any repercussions. Particular attention will be paid to any visible distress shown by the children, at which point they will be removed from the study environment and their data destroyed. All information will be held confidentially in accordance with the Data Protection Act and will be anonymously coded. Any parents/ guardians/ children wishing for data to be removed from the study may contact one of the researchers and their data will be destroyed.

If you **DO NOT** give permission for your child to participate, please return the following consent form to the school before [DATE]. If the consent form is not returned before this date, I will assume your permission has been granted for your child's participation.

For further information, or to ask any questions, contact details are provided below.

Yours sincerely,

Katie Allen

katie.allen@durham.ac.uk
s.e.higgins@durham.ac.uk (Professor Steve Higgins, Supervisor)

I **DO NOT** give permission for (name of child) ____________________ to participate in the above study

Name of parent/ guardian: ______________________

Parent/ guardian signature: __________________

Date: _________________

Appendix G: Permission letter for study 3

Dear parent/ guardian,

I am a postgraduate student at Durham University, studying for my PhD. For my project, I am investigating the relationship between working memory and the different elements of maths. I am hoping to test a group of children in Years 3 and 4 at [SCHOOL NAME] and request your permission for your child's participation in the study. The study complies with the ethical guidelines of the British Psychological Society (BPS) for psychological research and the British Education Research Association (BERA).

This work is a continuation of the research I completed with these children previously, and aims to explore whether it is possible to predict children's mathematics performance from their individual working memory profile. The children completed a mathematics measure previously so will not be required to do this again. Your child will be asked to complete subtests from a working memory battery. The complete test will take approximately 40 minutes, with the opportunity for them to withdraw at any point, should they feel they do not wish to continue, without giving reason and without any repercussions. Particular attention will be paid to any visible distress shown by the children, at which point they will be removed from the study environment and their data destroyed. All information will be held confidentially in accordance with the Data Protection Act and will be anonymously coded. Any parents/ guardians/ children wishing for data to be removed from the study may contact one of the researchers and their data will be destroyed.

If you **DO NOT** give permission for your child to participate, please return the following consent form to the school before [DATE]. If the consent form is not returned before this date, I will assume your permission has been granted for your child's participation.

For further information, or to ask any questions, contact details are provided below.

Yours sincerely,

Katie Allen

katie.allen@durham.ac.uk
s.e.higgins@durham.ac.uk (Professor Steve Higgins, Supervisor)

I **DO NOT** give permission for (name of child) ____________________ to participate in the above study

Name of parent/ guardian: ______________________

Parent/ guardian signature: __________________

Date: _________________

Appendix H: Permission letter for study 4
Dear parent/ guardian,

I am a postgraduate student at Durham University, studying for my PhD. For my project, I am investigating the relationship between working memory and the different elements of maths. I am conducting a 2 year follow up to my original study and am hoping to test the children in Year 5 at [SCHOOL NAME] so request your permission for your child's participation in the study. The study complies with the ethical guidelines of the British Psychological Society (BPS) for psychological research and the British Education Research Association (BERA).

This work is a continuation of the research I completed with these children previously, and aims to explore whether it is possible to predict children's mathematics performance from their earlier individual working memory profile. The children completed the working memory measures previously so will not be required to do this again. Your child will be asked to complete a single mathematics test. The complete test will take approximately 45 minutes, with the opportunity for them to withdraw at any point, should they feel they do not wish to continue, without giving reason and without any repercussions. Particular attention will be paid to any visible distress shown by the children, at which point they will be removed from the study environment and their data destroyed. All information will be held confidentially in accordance with the Data Protection Act and will be anonymously coded. Any parents/ guardians/ children wishing for data to be removed from the study may contact one of the researchers and their data will be destroyed.

If you **DO NOT** give permission for your child to participate, please return the following consent form to the school before [DATE]. If the consent form is not returned before this date, I will assume your permission has been granted for your child's participation.

For further information, or to ask any questions, contact details are provided below.

Yours sincerely,

Katie Allen

katie.allen@durham.ac.uk
s.e.higgins@durham.ac.uk (Professor Steve Higgins, Supervisor)

I **DO NOT** give permission for (name of child) ____________________ to participate in the above study

Name of parent/ guardian: ______________________

Parent/ guardian signature: __________________

Date: _________________

Appendix I: Title page of Allen, Higgins, et al. (2019)

Educational Psychology Review
https://doi.org/10.1007/s10648-019-09470-8

REVIEW ARTICLE

The Relationship between Visuospatial Working Memory and Mathematical Performance in School-Aged Children: a Systematic Review

Katie Allen[1] · Steve Higgins[1] · John Adams[2]

Published online: 08 February 2019

Abstract
The body of research surrounding the relationship between visuospatial working memory (VSWM) and mathematics performance remains in its infancy. However, it is an area generating increasing interest as the performance of school leavers comes under constant scrutiny. In order to develop a coherent understanding of the literature to date, all available literature reporting on the relationship between VSWM and mathematics performance was included in a systematic, thematic analysis of effect sizes. Results show a significant influence of the use of a standardised mathematics measure, however, no influence of the type of VSWM or mathematics being assessed, on the effect sizes generated. Crucially, the overall effect size is positive, demonstrating a positive association between VSWM and mathematics performance. The greatest implications of the review are on researchers investigating the relationship between VSWM and mathematics performance. The review also highlights as yet under-researched areas with scope for future research.

Keywords Visuospatial · Working memory · Mathematics performance · Numerical operations · Mathematical reasoning

Electronic supplementary material The online version of this article (https://doi.org/10.1007/s10648-019-09470-8) contains supplementary material, which is available to authorized users.

✉ Katie Allen
katie.allen@durham.ac.uk

1 School of Education, University of Durham, Durham, UK

2 Department of Psychology, University of Durham, Durham, UK

Springer

Appendix J: Title page of Allen et al. (2020c)

Original Article

QJEP

Quarterly Journal of Experimental Psychology
2020, Vol. 73(2) 239–248

DOI: 10.1177/1747021819871243
qjep.sagepub.com
SAGE

Working memory predictors of written mathematics in 7- to 8-year-old children

Katie Allen[1], David Giofrè[2], Steve Higgins[1] and John Adams[3]

Abstract

There is extensive evidence for the involvement of working memory in mathematical attainment. This study aims to identify the relative contributions of verbal, spatial-simultaneous, and spatial-sequential working memory measures in written mathematics. Year 3 children (7–8 years of age, $n = 214$) in the United Kingdom were administered a battery of working memory tasks alongside a standardised test of mathematics. Confirmatory factor analyses and variance partitioning were then performed on the data to identify the unique variance accounted for by verbal, spatial-simultaneous, and spatial-sequential measures. Results revealed the largest individual contribution was that of verbal working memory, followed by spatial-simultaneous factors. This suggests the components of working memory underpinning mathematical performance at this age are those concerning verbal-numeric and spatial-simultaneous working memory. Implications for educators and further research are discussed.

Keywords

Verbal; visuospatial; working memory; mathematics; children

Received: 3 December 2018; revised: 21 July 2019; accepted: 26 July 2019

Introduction

There is some discrepancy in the literature with regard to the proportional influence of components of the Baddeley and Hitch (1974) working memory model on mathematics achievement. While there are suggestions of a stronger influence of visuospatial working memory (VSWM; e.g., Caviola, Mammarella, Lucangeli, & Cornoldi, 2014; Clearman, Klinger, & Szűcs, 2017; Holmes, Adams, & Hamilton, 2008; Li & Geary, 2017), there is also evidence of developmental shifts in the respective contributions and the potential for a cyclical relationship (e.g., Li & Geary, 2013; Soltanlou, Pixner, & Nuerk, 2015; Van de Weijer-Bergsma, Kroesbergen, & Van Luit, 2015). In addition, there is some evidence for a greater influence of verbal working memory (e.g., Wilson & Swanson, 2001) on mathematics. VSWM is implicated in mathematics performance in a number of areas, including, but not limited to arithmetic (Ashkenazi, Rosenberg-Lee, Metcalfe, Swigart, & Menon, 2013; Caviola, Mammarella, Cornoldi & Lucangeli, 2012; Passolunghi & Cornoldi, 2008), word problem solving (Swanson & Beebe-Frankenberger, 2004; Swanson & Sachse-Lee, 2001; Zheng, Swanson, & Marcoulides, 2011) and geometry (Giofrè, Mammarella, & Cornoldi, 2014; Giofrè, Mammarella, Ronconi & Cornoldi, 2013), as well as mathematical difficulties (Andersson & Lyxell, 2007; D'Amico & Guarnera, 2005; McLean & Hitch, 1999; Passolunghi & Cornoldi, 2008; Szűcs, Devine, Soltesz, Nobes, & Gabriel, 2013). It is, therefore, important to understand the intricacies of this relationship to mediate difficulties associated with mathematics to the fullest extent possible.

Some authors argued that the VSWM system is not unitary (e.g., Logie, 2014). An alternative approach that has recently received some support is one that distinguishes between spatial-sequential tasks, requiring the recall of a sequence of spatial locations, and spatial-simultaneous tasks, demanding the recall of an array of simultaneously presented locations (see Mammarella,

[1]School of Education, Durham University, Durham, UK
[2]Department of Educational Sciences, University of Genoa, Genoa, Italy
[3]Department of Psychology, Durham University, Durham, UK

Corresponding author:
Katie Allen, School of Education, Durham University, Durham DH1 1TA, UK.
Email: katie.allen@durham.ac.uk

Appendix K: Title page of Allen et al. (2020b)

the british psychological society
promoting excellence in psychology

British Journal of Educational Psychology (2020)

www.wileyonlinelibrary.com

Working memory predictors of mathematics across the middle primary school years

Katie Allen[1]*, David Giofrè[2], Steve Higgins[3] and John Adams[4]

[1]School of Education, University of Durham, UK
[2]Dipartimento di Scienze della Formazione (DISFOR), University of Genoa, Italy
[3]School of Education, University of Durham, UK
[4]Department of Psychology, University of Durham, UK

Background. Work surrounding the relationship between visuospatial working memory (WM) and mathematics performance is gaining significant traction as a result of a focus on improving academic attainment.

Aims. This study examined the relative contributions of verbal and visuospatial simple and complex WM measures to mathematics in primary school children aged 6–10 years.

Sample. A sample of 111 children in years 2–5 were assessed (M_{age} = 100.06 months, SD = 14.47).

Method. Children were tested individually on all memory measures, followed by a separate mathematics testing session as a class group in the same assessment wave.

Results and Conclusions. Results revealed an age-dependent relationship, with a move towards visuospatial influence in older children. Further analyses demonstrated that backward word span and backward matrices contributed unique portions of variance of mathematics, regardless of the regression model specified. We discuss possible explanations for our preliminary findings in relation to the existing literature alongside their implications for educators and further research.

There is an increasing wealth of literature on the relationship between working memory (WM) and academic attainment in school-aged children. WM can be operationally defined as the capacity to temporarily store and manipulate information, necessary for the completion of complex tasks (Baddeley, 1992). The model of WM proposed by Baddeley and Hitch (1974) has been developed since its conception to include two slave systems, the visuospatial sketchpad and the phonological loop, responsible for the storage and manipulation of visual and verbal information, respectively (Baddeley, 2000). The visuospatial sketchpad, therefore, supports visuospatial WM, while the phonological loop supports verbal WM. This WM model continues to be robust to methodological advances and research findings, and has repeatedly been used in studies conducted with typically

Correspondence should be addressed to Katie Allen, School of Education, University of Durham, Leazes Road, Durham, UK (email: katie.allen@durham.ac.uk).

DOI:10.1111/bjep.12339

Appendix L: Title page of Allen, Giofrè, Higgins and Adams (2020a)

Psychological Research
https://doi.org/10.1007/s00426-020-01382-5

ORIGINAL ARTICLE

Using working memory performance to predict mathematics performance 2 years on

Katie Allen[1] · David Giofrè[2] · Steve Higgins[1] · John Adams[3]

Received: 31 January 2020 / Accepted: 1 July 2020

Abstract
A number of previous studies have used working memory components to predict mathematical performance in a variety of ways; however, there is no consideration of the contributions of the subcomponents of visuospatial working memory to this prediction. In this paper we conducted a 2-year follow-up to the data presented in Allen et al. (Q J Exp Psychol 73(2):239–248, 2020b) to ascertain how these subcomponents of visuospatial working memory related to later mathematical performance. 159 children (M age = 115.48 months) completed the maths test for this second wave of the study. Results show a shift from spatial–simultaneous influence to spatial–sequential influence, whilst verbal involvement remained relatively stable. Results are discussed in terms of their potential for education and future research.

Introduction

Using working memory to predict mathematical attainment is an area of study that has gained a significant amount of traction in recent years. Mathematics is a broad field and there has been extensive research across a number of aspects of mathematics and working memory which has been summarised in reviews and meta-analyses, from studies of typically developing populations (Friso-van den Bos et al., 2013; Raghubar et al., 2010), to the relationship with learning difficulties in mathematics generally (David, 2012; Swanson & Jerman, 2006) and in terms of the verbal and numerical domains in particular (Peng & Fuchs, 2016). According to the multicomponent model (Baddeley & Hitch, 1974), working memory involves subcomponents relating to the processing of visuospatial and phonological stimuli. The components of working memory have been reliably linked to academic performance on a number of occasions (e.g., Alloway & Passolunghi, 2011; Holmes & Adams, 2006; Van de Weijer-Bergsma, Kroesbergen, & Van Luit, 2015; see Peng, Namkung, Barnes, & Sun, 2016 for a review of this literature) with a reasonable amount of evidence suggesting visuospatial working memory is more influential in younger children (e.g., Caviola, Mammarella, Lucangeli, & Cornoldi, 2014; Clearman, Klinger, & Szucs, 2017; Holmes, Adams, & Hamilton, 2008). There is also a smaller, though not insignificant, amount of evidence indicating the involvement of verbal working memory (e.g., Kyttälä, Kanerva, Munter, & Björn, 2019; Wilson & Swanson, 2001); a finding we replicated at time 1 (T1) of this study (Allen, Giofrè, Higgins & Adams, 2020b).

At T1, results revealed that, when compared directly to spatial–simultaneous and spatial–sequential measures, verbal-numeric tasks were more predictive of mathematics in 7–8-year-old children. Similarly, Allen, Giofrè, Higgins and Adams (2020a) demonstrated that verbal working memory (non-numeric) was more predictive of mathematical performance in younger children, with a move toward visuospatial influence in older children. It is not yet fully understood, however, how these components relate specifically to mathematical attainment on a longitudinal basis. There is some evidence suggesting visuospatial working memory is influential in the prediction of mathematics over a number of years (e.g., Bull, Espy, & Wiebe, 2008; De Smedt et al., 2009; Fanari, Meloni, & Massidda, 2019; Geary, 2011; Hilbert, Bruckmaier, Binder, Krauss, & Bühner, 2019; Li & Geary, 2017); however, as indicated by Hilbert et al. (2019), it is necessary to consider the mathematics test used for the purposes of these studies. In some

✉ Katie Allen
katie.allen@durham.ac.uk

[1] School of Education, University of Durham, Durham, UK

[2] Dipartimento di Scienze della Formazione (DISFOR), University of Genoa, Genoa, Italy

[3] Department of Psychology, University of Durham, Durham, UK

Published online: 10 July 2020

Springer